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PRODUCTION OF HEAVY QUARKONIUM IN HIGH ENERGY COLLIDERS

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Abstract

Recent data from the Tevatron has revealed that the production rate of prompt charmonium at large transverse momentum is orders of magnitude larger than the best theoretical predictions of a few years ago. These surprising results can be understood by taking into account two recent developments that have revolutionized the theoretical description of heavy quarkonium production. The first is the realization that fragmentation must dominate at large transverse momentum, which implies that most charmonium in this kinematic region is produced by the hadronization of individual high- p_T partons. The second is the development of a factorization formalism for quarkonium production based on non-relativistic QCD that allows the formation of charmonium from color-octet $c\bar{c}$ pairs to be treated systematically. This review summarizes these theoretical developments and their implications for quarkonium production in high energy colliders.

1 INTRODUCTION

The goal of high energy physics is to identify the elementary constituents of matter and to understand their fundamental interactions. Over the last twenty years, this endeavor has been extraordinarily successful. A gauge theory called the minimal Standard Model provides a satisfactory description of the strong, weak, and electromagnetic interactions of all the known elementary particles. There are very few discrepancies between theory and experiment, and most of them are at the level of a few

standard deviations or less. However there is one process for which experimental results have differed from theoretical predictions by orders of magnitude: the production of charmonium at large transverse momentum at the Tevatron. This dramatic conflict between experiment and theory presents a unique opportunity to make a significant step forward in our understanding of heavy quarkonium physics.

Prior to 1993, the conventional wisdom on the production of charmonium in hadron collisions was based primarily on calculations in the *color-singlet model*. In this model, the production of a charmonium state is assumed to proceed through parton processes that produce a $c\bar{c}$ pair in a color-singlet state. It is the predictions of the color-singlet model at lowest order in α_s that disagree so dramatically with the Tevatron data. The data can, however, be explained by combining two recent theoretical developments in heavy quarkonium physics. The first is the realization that heavy quarkonium at large transverse momentum is produced primarily by *fragmentation*, the hadronization of individual high- p_T partons. In the color-singlet model, fragmentation first contributes at higher order in α_s and thus was not taken into account in previous calculations. The second development is the realization that *color-octet mechanisms*, in which the $c\bar{c}$ pair is produced at short distances in a color-octet state, sometimes dominate the production of charmonium. These mechanisms can be analyzed systematically using a new factorization formalism for heavy quarkonium production that is based on an effective field theory called *nonrelativistic QCD* (NRQCD). The Tevatron data can be explained by including a color-octet term in the fragmentation function for the formation of charmonium in a gluon jet. Evidence in support of this explanation has been accumulating, but a great deal of work, both experimental and theoretical, will be required before it can be regarded as conclusive.

The purpose of this review is to summarize the recent theoretical developments in heavy quarkonium production. A brief review of these developments has been given previously by Mangano [1]. Our main focus will be on the production of quarkonium in high energy colliders, where both fragmentation and color-octet mechanisms are important. The color-singlet model is reviewed in Section 2. We summarize its predictions for charmonium production in $p\bar{p}$ colliders, which are in dramatic disagreement with Tevatron data. The production of quarkonium by fragmentation is reviewed in Section 3. We describe the factorization theorems of perturbative QCD that imply that fragmentation should dominate at large transverse momentum, and we illustrate them using the electromagnetic production of the J/ψ in Z^0 decay. We then

discuss the predictions of the color-singlet model for fragmentation functions. Color-octet mechanisms for producing quarkonium are reviewed in Section 4. We first discuss the production of P-wave states, where a color-octet mechanism is required for perturbative consistency. We then describe the NRQCD factorization formalism, which implies that color-octet mechanisms must also contribute to the production of S-wave states. We also discuss its implications for fragmentation functions. In Section 5, we summarize applications of the recent theoretical developments to the production of prompt charmonium, bottomonium, and the B_c at the Tevatron and at LEP. We conclude in Section 6 with an outlook on the work that will be required to develop a comprehensive understanding of heavy quarkonium production in high energy processes.

2 THE PROBLEM OF CHARMONIUM PRODUCTION

Before 1993, most predictions for charmonium production were based on the *color-singlet model*. In this Section, we review the color-singlet model and summarize its predictions at leading order in α_s for charmonium production at large transverse momentum in $p\bar{p}$ collisions. We then describe the experimental results from the Tevatron that have forced a reexamination of the problem of charmonium production.

2.1 *Color-singlet Model*

It is difficult to ascribe credit for the *color-singlet model*, since many physicists were involved in its early development. The decays of B mesons into charmonium states through the process $b \rightarrow c\bar{c} + s$ was first treated in the color-singlet model by DeGrand and Toussaint, by Wise, and by Kühn, Nussinov, and Rückl [2]. The hadronic production of J/ψ through the parton process $gg \rightarrow c\bar{c} + g$ was calculated by Chang [3]. A thorough treatment of charmonium production in hadron collisions through $2 \rightarrow 3$ parton processes was later presented by Baier and Rückl [4]. Guberina, Kühn, Peccei, and Rückl [5] applied the color-singlet model to charmonium production from the decay $Z^0 \rightarrow c\bar{c} + \gamma$. Berger and Jones [6] calculated the rate for photoproduction of charmonium, and emphasized the kinematical restrictions on the applicability of the color-singlet model. This model was also applied to the inclusive production of charmonium in e^+e^- annihilation by Keung and by Kühn

and Schneider [7]. A thorough review of the applications of the color-singlet model to heavy quarkonium production was recently given by Schuler [8].

An alternative model for quarkonium production called the *color-evaporation model* was developed around the same time [9]. In this model, it was assumed that all $c\bar{c}$ pairs with invariant mass between $2m_c$ and the $D\bar{D}$ threshold $2m_D$ produce charmonium states. The fraction f_H of the $c\bar{c}$ pairs that form a particular charmonium state H is assumed to be independent of the production process. This model is incapable of describing the variation of the production ratios for charmonium states between processes and as functions of kinematical variables. It will therefore not be considered further.

To motivate the color-singlet model, we can think of the production of charmonium as proceeding in two steps. The first step is the production of a $c\bar{c}$ pair, and the second step is the binding of the $c\bar{c}$ pair into a charmonium state. In order to have a significant probability of binding, the $c\bar{c}$ pair must be produced with relative momentum that, in the $c\bar{c}$ rest frame, is small compared to the mass m_c of the charm quark. Otherwise, the c and \bar{c} will fly apart and ultimately form D and \bar{D} mesons.

We first consider the production of the $c\bar{c}$ pair. Assuming that the c and \bar{c} are not present in the initial state, any Feynman diagram for the production of a $c\bar{c}$ pair must involve virtual particles that are off their mass shells by amounts of order m_c or larger. The part of the amplitude in which all internal lines are off-shell by amounts of order m_c or larger is called the *short-distance* part, and it is calculable using perturbation theory in $\alpha_s(m_c)$. The parts of the amplitude in which the c and \bar{c} lines are off-shell by amounts much less than m_c can be considered part of the amplitude for the formation of the bound state. The short-distance part of the amplitude describes the production of a $c\bar{c}$ pair with a spatial separation that is of order $1/m_c$ or smaller. This follows from the fact that the short-distance part is insensitive to changes in the relative 3-momentum of the c and \bar{c} that are much less than m_c . Since $1/m_c$ is much smaller than the length scale associated with the charmonium wavefunction, the $c\bar{c}$ pair is essentially pointlike on that scale. Thus we need only consider the amplitude for a pointlike $c\bar{c}$ pair to bind to form a charmonium state. This amplitude will necessarily depend on the charmonium state H and on the quantum numbers of the $c\bar{c}$ pair.

For any given charmonium state, the dominant Fock state consists of a color-singlet $c\bar{c}$ pair in a definite angular-momentum state. We denote the two possible color states of a $c\bar{c}$ pair by $\underline{1}$ for color-singlet and $\underline{8}$ for

color-octet. We use the spectroscopic notation $^{2S+1}L_J$ for the angular momentum state, where S , L , and J are the quantum number associated with the total spin, the orbital angular momentum, and the total angular momentum, respectively. Thus the dominant Fock state for a charmonium state H is denoted $|c\bar{c}(\underline{1}, ^{2S+1}L_J)\rangle$ for some appropriate values of S , L , and J . For example, the dominant Fock state for the J/ψ is $|c\bar{c}(\underline{1}, ^3S_1)\rangle$, while for the χ_{cJ} , it is $|c\bar{c}(\underline{1}, ^3P_J)\rangle$.

The *color-singlet model* is a simple model for the amplitudes for a pointlike $c\bar{c}$ pair to form a charmonium state. If the dominant Fock state of the meson H is $|c\bar{c}(\underline{1}, ^{2S+1}L_J)\rangle$, the amplitude is assumed to be 0 unless the pointlike $c\bar{c}$ pair is in a color-singlet $^{2S+1}L_J$ state. For this state, the amplitude can be expressed in terms of the L 'th derivative of the radial wavefunction at the origin for the meson H . For example, the amplitudes for producing the states ψ and χ_{cJ} , plus some specific final state F , are assumed to have the forms

$$\mathcal{A}(\psi + F) = \hat{\mathcal{A}}(c\bar{c}(\underline{1}, ^3S_1) + F) R_\psi(0), \quad (1)$$

$$\mathcal{A}(\chi_{cJ} + F) = \hat{\mathcal{A}}(c\bar{c}(\underline{1}, ^3P_J) + F) R'_{\chi_c}(0). \quad (2)$$

The $\hat{\mathcal{A}}$'s are amplitudes for producing color-singlet $c\bar{c}$ pairs with vanishing relative momentum in the angular-momentum states indicated. The factor $R_\psi(0)$ is the radial wavefunction at the origin for the ψ , while $R'_{\chi_c}(0)$ is the derivative of the radial wavefunction at the origin for the χ_c states. In the color-singlet model, it is assumed that the amplitudes \mathcal{A} can be calculated using perturbative QCD and that all nonperturbative effects can be absorbed into the wavefunction factors. From (1) and (2), we deduce that the corresponding inclusive differential cross sections in the color-singlet model have the forms

$$d\sigma(\psi + X) = d\hat{\sigma}(c\bar{c}(\underline{1}, ^3S_1) + X) |R_\psi(0)|^2, \quad (3)$$

$$d\sigma(\chi_{cJ} + X) = d\hat{\sigma}(c\bar{c}(\underline{1}, ^3P_J) + X) |R'_{\chi_c}(0)|^2. \quad (4)$$

The color-singlet model has enormous predictive power. The cross section for producing a quarkonium state in any high energy process is predicted in terms of a single nonperturbative parameter for each orbital-angular-momentum multiplet. For example, the nonperturbative factor is $R_\psi(0)$ for the S-wave states ψ and η_c . It is $R'_{\chi_c}(0)$ for the P-wave states χ_{c0} , χ_{c1} , χ_{c2} , and h_c . Moreover, these parameters can be determined from decays of the charmonium states. For example, $R_\psi(0)$ can be determined from the electronic width of the ψ :

$$\Gamma(\psi \rightarrow e^+e^-) \approx \frac{4\alpha^2}{9m_c^2} |R_\psi(0)|^2. \quad (5)$$

Thus the color-singlet model gives absolutely normalized predictions for the production rates of charmonium states in high energy collisions.

In spite of its great predictive power, the color-singlet model is only a model. There are no theorems that guarantee that the amplitude factors in the simple way assumed in (1) and (2). In particular, it was never proven that higher order radiative corrections would respect the factored form. In addition, the model is clearly incomplete. For one thing, relativistic corrections, which take into account the relative velocity v of the quark and antiquark, are neglected. These corrections are probably not negligible for charmonium, since the average value of v^2 is only about $1/3$. The color-singlet model also assumes that a $c\bar{c}$ pair produced in a color-octet state will never bind to form charmonium. This assumption must break down at some level, since a color-octet $c\bar{c}$ pair can make a nonperturbative transition to a color-singlet state by radiating a soft gluon. The most glaring evidence that the color-singlet model is incomplete comes from the presence of infrared divergences in the cross sections for P-wave states. This problem and its solution will be discussed in Section 4. For the moment, we simply note that the infrared divergence violates the factorization assumption implicit in (4). It implies that the cross section $d\hat{\sigma}$ is sensitive to small momentum scales, so it cannot be calculated reliably using perturbative QCD.

2.2 *Expectations for Charmonium at Large p_T*

Most calculations of charmonium production prior to 1993 were based on two crucial assumptions. The first was that the amplitude for the formation of a charmonium state from a $c\bar{c}$ pair was accurately described by the color-singlet model. The second was that the dominant production processes for color-singlet $c\bar{c}$ pairs were the Feynman diagrams that were lowest order in α_s . The first thorough treatment of the problem of charmonium production at large transverse momentum in hadron-hadron collisions was given by Baier and Rückl in 1983 [4]. In subsequent calculations [10], the contributions from B -meson decay into charmonium states were included. The results of these calculations are summarized below.

We first introduce some terminology for describing charmonium production in high energy colliders. Charmonium is called *prompt* if the point at which the charmonium state is produced and the collision point of the colliding beams can not be resolved using a vertex detector. Prompt charmonium is produced by QCD production mechanisms. The decay of b -hadrons produces charmonium that is not prompt. If

a b -hadron is produced with large transverse momentum, it will travel a significant distance before decaying weakly. For a b -hadron with p_T around 10 GeV, the displacement between the collision point and the secondary vertex where the charmonium is produced is typically a fraction of a millimeter.

A charmonium state that is prompt, but does not come from the decay of a higher charmonium state, is called *direct*. For example, the ψ is produced in the decays of χ_{c0} , χ_{c1} , χ_{c2} , and ψ' with branching fractions of approximately 0.7%, 27%, 14%, and 57%, respectively. Thus the prompt ψ signal includes direct ψ 's and contributions from direct χ_{cJ} and direct ψ' .

A charmonium state with large momentum is called *isolated* if there are no other hadrons whose momentum is nearly collinear. Charmonium that is produced from the decay of a b -hadron with large transverse momentum is never isolated, because the remnant hadrons from the decay of the b -hadron will have momentum that is nearly collinear.

We now consider the mechanisms for the production of charmonium with large p_T in $p\bar{p}$ collisions. First we focus on charmonium production from the weak decay of b -hadrons. Note that the decay of b -hadrons with large p_T produces charmonium that is neither prompt nor isolated. The dominant mechanism for producing b -hadrons at large p_T is the gluon fusion process $gg \rightarrow b\bar{b}$, followed by the hadronization of the b or \bar{b} . The inclusive branching fractions for any b -hadron to decay into charmonium states are presumably close to those for B mesons. The inclusive branching fractions for B -meson decays are approximately 1.3% for ψ , 1% for χ_{c1} , and 0.5% for ψ' .

The color-singlet model gives predictions for the production rate of charmonium from QCD mechanisms. If a $c\bar{c}$ pair is produced with large p_T through parton collisions, there must be a recoiling parton to balance the transverse momentum. Thus the production mechanisms that are of leading order in α_s are $2 \rightarrow 3$ parton processes. Note that the charmonium states produced by these processes are prompt and isolated. In the color-singlet model, the only $2 \rightarrow 3$ process that can produce ψ or ψ' is $gg \rightarrow c\bar{c} + g$. The $2 \rightarrow 3$ processes that produce χ_{cJ} 's with large p_T are $gg \rightarrow c\bar{c} + g$, $gq \rightarrow c\bar{c} + q$, $g\bar{q} \rightarrow c\bar{c} + \bar{q}$, and $q\bar{q} \rightarrow c\bar{c} + g$. The cross sections for these $2 \rightarrow 3$ processes are all of order α_s^3 . At large p_T , the parton differential cross sections $d\hat{\sigma}/dp_T^2$ scale like $1/p_T^6$ for the χ_{cJ} states and like $1/p_T^8$ for ψ and ψ' . Thus the parton cross sections for ψ and ψ' are suppressed relative to those for χ_{cJ} by a factor of m_c^2/p_T^2 at large p_T .

We now summarize the predictions of the color-singlet model at lead-

ing order in α_s for inclusive ψ production in $p\bar{p}$ collisions [10]. The contribution from b -hadron decay falls off the most slowly with p_T and it was predicted to dominate at the Tevatron for $p_T > 7$ GeV. Of the prompt production mechanisms for ψ , the most important was found to be the decay of direct χ_{c1} . The contributions from direct χ_{c2} and direct ψ were down by factors of about 4 and 18 at $p_T = 10$ GeV. Thus the conventional wisdom before 1993 was that ψ production at large p_T at the Tevatron should be dominated by b -hadron decay, with decays of direct χ_{c1} and direct χ_{c2} being the only other important mechanisms.

In the case of ψ' , the decay of b -hadrons was predicted to be the only important production mechanism at large p_T . It should be emphasized that these predictions were based on the color-singlet model and on the additional assumption that the dominant parton processes were of order α_s^3 .

2.3 Prompt Charmonium at the Tevatron

The first substantial data on the production of charmonium in $p\bar{p}$ collisions came from the $S\bar{p}\bar{p}$ S operating at a center-of-mass energy of 630 GeV. Data collected by the UA1 collaboration contained indications of deviations from the predictions of the color-singlet model [11]. The Tevatron, which operates at the significantly higher energy of 1.8 TeV, has provided an opportunity to investigate charmonium production in much greater detail. The CDF collaboration has accumulated large samples of data on the production of ψ , χ_{cJ} , and ψ' at the Tevatron. Though analysis of the data from the 1988-1989 collider run was hampered by the inability to separate prompt charmonium from charmonium that is produced by the decays of b -hadrons, discrepancies between the data and the predictions of the color-singlet model were evident [12]. However, assuming the conventional wisdom that ψ production at large p_T should be dominated by the decays of b -hadrons and direct χ_c 's, CDF extracted a value for the b quark cross section from their data on ψ and χ_c production [13]. This value was shown to be too large by about a factor of 2 in the subsequent collider run [14].

Before the 1992-93 run of the Tevatron, CDF installed a silicon vertex detector that can resolve secondary vertices separated by distances greater than about $10 \mu\text{m}$ from the $p\bar{p}$ collision point [15]. Prompt charmonium is produced essentially at the collision point, while charmonium from the decay of b -hadrons is produced at a secondary vertex which is typically hundreds of microns away. Thus the silicon vertex detector can be used to separate prompt charmonium from charmonium that is

produced in b -hadron decays.

In the charmonium data sample collected during the 1992-93 run, it was found that only 20% of the ψ 's and only 23% of the ψ' 's come from the decay of b -hadrons [16]. Furthermore, only 32% of the ψ 's come from χ_c decay [17]. Thus, the majority of the ψ 's and ψ' 's must be produced by other mechanisms, in dramatic contradiction to the conventional wisdom. Furthermore, the fraction of ψ and ψ' from b -hadron decay does not increase appreciably with increasing p_T , contrary to the predictions of the color-singlet model.

An excellent review of the results on quarkonium production from the Tevatron, and also from other experiments, has been presented by Sansoni [18]. CDF has measured the cross sections for prompt ψ and prompt ψ' production as a function of p_T [16]. The prompt ψ signal has been resolved into those ψ 's that come from χ_c decays and those that do not [17]. In addition, the ratio of the production rates of χ_{c1} and χ_{c2} has been measured. The most recent CDF data on the production of prompt ψ 's that do not come from χ_c decay, prompt ψ 's from χ_c decay, and prompt ψ' 's, are shown in Figures 1, 2, and 3, respectively. The prompt ψ 's that do not come from χ_c decay include direct ψ 's and ψ 's from the decay of direct ψ' 's. The predictions of the color-singlet model at leading order in α_s are shown as dashed lines. These predictions fall several orders of magnitude below the data at large p_T . Thus the predictions of the color-singlet model at lowest order in α_s fail dramatically when confronted with the data on charmonium production from the Tevatron.

3 FRAGMENTATION

The first major conceptual advance in the recent revolution in heavy quarkonium production was the realization that *fragmentation* dominates at sufficiently large transverse momentum. Fragmentation is the formation of a hadron within a jet produced by a parton (quark, anti-quark, or gluon) with large transverse momentum. As the parton emerges from the collision point, it radiates gluons and other partons, most of which are almost collinear. The partons ultimately coalesce into the hadrons that make up the jet. In the case of production of charmonium by fragmentation, the “jet” containing the charmonium state may not qualify as a jet by conventional experimental definitions. For example, if most of the momentum of the “jet” is carried by the charmonium state, it may not satisfy a jet criterion that requires a specified number

of tracks above a certain momentum threshold.

The word *fragmentation* is sometimes used to describe the coalescence of partons into hadrons, whether or not these partons make up a jet. We prefer to use the word *hadronization* to describe this general process. The word *fragmentation* will be reserved specifically for the formation of a hadron within the jet produced by a high- p_T parton. Thus fragmentation involves the hadronization of the partons in the jet, but hadronization also occurs in low energy processes that have nothing to do with jets. Fragmentation is a useful concept because the probability for the formation of a hadron within a jet is universal, *i.e.* it is independent of the process that produces the parton that initiates the jet.

In this Section, we introduce the factorization theorems of perturbative QCD that guarantee that inclusive hadron production at large transverse momentum is dominated by fragmentation. We illustrate the factorization theorems using the simple example of the electromagnetic production of ψ in Z^0 decay. Finally we discuss the color-singlet model predictions for the fragmentation functions of heavy quarkonium.

3.1 Factorization Theorems of Perturbative QCD

One of the classic factorization theorems of perturbative QCD [19] guarantees that inclusive hadron production in e^+e^- annihilation at sufficiently large energies is dominated by fragmentation. After describing this factorization theorem, we discuss its extension to inclusive hadron production at large transverse momentum in hadron-hadron collisions.

We consider the inclusive production of a hadron H with energy E in e^+e^- annihilation at large center-of-mass energy \sqrt{s} . We are interested in the cross section in the *scaling limit* in which $E, \sqrt{s} \rightarrow \infty$ with E/\sqrt{s} held fixed. The production of the hadron also involves lower momentum scales, such as the hadron mass and the scale Λ_{QCD} associated with nonperturbative effects in QCD. The factorization theorem states that the cross section in the scaling limit has the form [19]

$$\begin{aligned} d\sigma(e^+e^- \rightarrow H(E) + X) \\ = \sum_i \int_0^1 dz d\hat{\sigma}(e^+e^- \rightarrow i(E/z) + X, \mu) D_{i \rightarrow H}(z, \mu), \end{aligned} \quad (6)$$

where the sum is over parton types i and the integral is over the longitudinal momentum fraction z of the hadron H relative to the parton i . In (6), $d\hat{\sigma}$ is the differential cross section for producing a parton i with

total energy E/z . This cross section is only sensitive to momenta on the order of E , so it can be calculated using QCD perturbation theory. The effects of lower momentum scales can be systematically factored into the functions $D_{i \rightarrow H}(z, \mu)$, which are called *fragmentation functions* or *parton decay functions*. The fragmentation function $D_{i \rightarrow H}(z, \mu)$ gives the probability that the jet initiated by parton i will include a hadron H carrying a fraction z of the jet momentum. The factorization theorem holds to all orders in perturbation theory. For a light hadron H whose mass is on the order of Λ_{QCD} , all corrections to the factorization formula (6) fall like powers of Λ_{QCD}/E .

The essential ingredient in the proof of the factorization theorem was given by Collins and Stermann [20]. They demonstrated that a diagram that contributes to the inclusive cross section in the scaling limit can be separated into a hard-scattering subdiagram that produces hard partons, jet-like subdiagrams for each of the hard partons, and a soft part. The soft part includes soft gluon lines that can couple to any of the jet-like subdiagrams. After summing over all possible connections of the soft gluons, one finds that the effects of the soft parts cancel, leaving a factored form for this contribution to the inclusive cross section. In the case of inclusive hadron production, these ideas were used by Curci, Furmanski, and Petronzio and by Collins and Soper [21] to provide field theoretic definitions of the fragmentation functions.

The factorization theorem (6) requires the introduction of an arbitrary scale μ that separates the large momentum scale E from the lower momentum scales. The parton cross sections and the fragmentation functions depend on the arbitrary scale μ in such a way that the cross section is independent of μ . The μ -dependence of the fragmentation functions is given by an evolution equation of the form

$$\mu^2 \frac{\partial}{\partial \mu^2} D_{i \rightarrow H}(z, \mu) = \sum_j \int_z^1 \frac{dy}{y} P_{i \rightarrow j}(z/y, \mu) D_{j \rightarrow H}(y, \mu). \quad (7)$$

The kernel $P_{i \rightarrow j}(x, \mu)$ describes the splitting of a parton i into a parton j with momentum fraction x , and is calculable as a perturbation series in $\alpha_s(\mu)$. At leading order in α_s , these kernels are identical to the Altarelli-Parisi functions that govern the evolution of parton distributions.

The factorization theorem for inclusive hadron production in e^+e^- annihilation can be generalized to other high energy processes. The real power of these factorization theorems lies in the fact that the fragmentation functions are universal, *i.e.* they are independent of the process that produces the fragmenting partons. Thus, if the fragmentation functions

for a hadron H are determined from e^+e^- annihilation data, they can be used to predict the production rate of the hadron H in jets produced by other high energy processes.

One of the important generalizations of the factorization theorem is to inclusive hadron production at large transverse momentum p_T in hadron-hadron collisions. The proof of the factorization theorem for this process is more difficult, because there are two hadrons in the initial state. In fact, such a proof has actually been carried out only for the simpler case of the Drell-Yan process for creating a muon pair [22]. However there are no apparent obstacles to extending this proof to the case of inclusive hadron production at large p_T [19]. The resulting factorization formula is

$$\begin{aligned} d\sigma(AB \rightarrow H(p_T) + X) &= \sum_{ijk} \int_0^1 dx_1 f_{j/A}(x_1) \int_0^1 dx_2 f_{k/B}(x_2) \\ &\times \int_0^1 dz d\hat{\sigma}(jk \rightarrow i(p_T/z) + X) D_{i \rightarrow H}(z). \end{aligned} \quad (8)$$

Long-distance effects can be factored into the fragmentation functions and into the parton distributions $f_{j/A}(x_1)$ and $f_{k/B}(x_2)$ for hadrons A and B . The restriction to large p_T is necessary in order that the jets consisting of the remnants of hadrons A and B after the hard scattering have large momentum relative to the jet containing hadron H . The formula (8) should hold to all orders in perturbation theory, with corrections falling like powers of Λ_{QCD}/p_T . Implicit in (8) are three arbitrary scales: the factorization scale μ_F , which cancels between the parton distributions and $d\hat{\sigma}$, the fragmentation scale μ_{frag} , which cancels between $d\hat{\sigma}$ and the fragmentation functions, and the renormalization scale μ_R for the running coupling constant, which appears in $d\hat{\sigma}$. In low-order calculations, μ_{frag} , μ_F , μ_R should all be chosen on the order of p_T/z , the transverse momentum of the fragmenting parton.

In 1993, Braaten and Yuan [23] pointed out that the factorization theorems for inclusive hadron production must apply to heavy quarkonium as well as to light hadrons. The only difference is that the leading corrections fall as powers of m_Q/p_T , where m_Q is the heavy quark mass, instead of powers of Λ_{QCD}/p_T . Therefore the dominant production mechanism for charmonium at $p_T \gg m_c$ must be fragmentation. In retrospect, this statement may seem obvious, but fragmentation contributions were not included in any of the previous calculations of charmonium production in $p\bar{p}$ collisions summarized in Section 2.2. This can easily be seen from the p_T dependence of the parton cross sections. In

the factorization formula (8), the only momentum scale that the parton cross section $d\hat{\sigma}$ can depend on is p_T . Therefore, by dimensional analysis, $d\hat{\sigma}/dp_T^2$ scales like $1/p_T^4$ at large p_T . The color-singlet model cross sections at leading order in α_s fall off much more rapidly with p_T : $d\hat{\sigma}/dp_T^2$ scales like $1/p_T^8$ for ψ and ψ' and like $1/p_T^6$ for χ_{cJ} .

3.2 Electromagnetic Production of ψ in Z^0 Decay

To illustrate the factorization theorem, we discuss the production of ψ in Z^0 decay through electromagnetic interactions. This example illustrates two important points. First, the production process that is lowest order in the coupling constant does not necessarily dominate in the asymptotic region. Second, the terms that do dominate in the asymptotic region can be factored into cross sections for producing partons and fragmentation functions.

The electromagnetic production of ψ is particularly simple, because the QCD interaction can be treated nonperturbatively by expressing the production amplitude in terms of a matrix element of the electromagnetic current between the QCD vacuum and a ψ :

$$\langle\psi|\frac{2}{3}\bar{c}\gamma^\mu c|0\rangle \equiv g_\psi M_\psi^2 \epsilon^\mu, \quad (9)$$

where ϵ^μ is the polarization vector for the ψ and g_ψ is dimensionless. The value of g_ψ can be determined from the decay rate for $\psi \rightarrow e^+e^-$:

$$\Gamma(\psi \rightarrow e^+e^-) = \frac{4\pi}{3}\alpha^2 M_\psi g_\psi^2. \quad (10)$$

This gives the value $g_\psi^2 = 0.008$.

The various contributions to electromagnetic ψ production in Z^0 decay can be calculated using perturbation theory in the electromagnetic coupling constant α . Since this coupling constant is small, we might naively expect the dominant production process to be the one that is lowest order in α . We must remember, however, that there is also another small dimensionless parameter in this problem, namely M_ψ/M_Z . The relative importance of a production process will be determined both by its order in α and by how it scales with M_ψ/M_Z . We will see that the leading-order process is suppressed by M_ψ^2/M_Z^2 , and that the dominant production process is actually higher order in α .

The production process for ψ that is lowest order in α is $Z^0 \rightarrow \psi\gamma$, which proceeds at order α^2 through the process $Z^0 \rightarrow c\bar{c} + \gamma$. For the $c\bar{c}$ to form a ψ , the relative momentum of the c and \bar{c} in the ψ rest frame must be small compared to m_c . This is in turn small compared

to the momentum of the ψ , which is approximately $M_Z/2$. If the relative momentum of the c and \bar{c} is neglected, the amplitude is proportional to the matrix element $\langle\psi|\bar{c}\gamma^\mu c|0\rangle$, and it therefore can be expressed in terms of g_ψ . The branching fraction for $Z^0 \rightarrow \psi\gamma$ is 5.2×10^{-8} [5]. In the limit $M_\psi \ll M_Z$, the branching fraction is proportional to $\alpha g_\psi^2 M_\psi^2/M_Z^2$, so it vanishes in the scaling limit. The suppression factor M_ψ^2/M_Z^2 reflects the fact that the c or \bar{c} must receive a momentum kick of order M_Z from the photon in order for the c and \bar{c} to be produced with small enough relative momentum to form a bound state.

At next-to-leading order in α , the ψ can be produced electromagnetically via the decay $Z^0 \rightarrow \psi + \ell^+ \ell^-$, which proceeds at order α^3 through the process $Z^0 \rightarrow c\bar{c} + \ell^+ \ell^-$. The branching fraction for $Z^0 \rightarrow \psi + \ell^+ \ell^-$ is 7.5×10^{-7} [24, 25]. Thus the decay rate for this order- α^3 process is an order of magnitude larger than the order- α^2 process $Z^0 \rightarrow \psi\gamma$. The reason it is larger is that the factor of M_ψ^2/M_Z^2 in the decay rate for $Z^0 \rightarrow \psi\gamma$ provides larger suppression than the extra factor of α in the decay rate for $Z^0 \rightarrow \psi + \ell^+ \ell^-$.

We introduce a *scaling limit* for the production of ψ with energy E in Z^0 decay. This limit is $E, M_Z \rightarrow \infty$ with E/M_Z and M_ψ held fixed. In the scaling limit, the differential decay rate for the electromagnetic production of ψ satisfies a factorization theorem that is analogous to that in (6):

$$\begin{aligned} d\Gamma(Z^0 \rightarrow \psi(E) + X) \\ = \sum_i \int_0^1 dz \, d\hat{\Gamma}(Z^0 \rightarrow i(E/z) + X, \mu) D_{i \rightarrow \psi}(z, \mu), \end{aligned} \quad (11)$$

where the sum over partons i includes photons and positive and negative leptons. The factor $d\hat{\Gamma}$ is the inclusive rate for decay into a parton i with energy E/z . The factor $D_{i \rightarrow \psi}(z)$ is a fragmentation function that gives the probability for an electromagnetic jet initiated by the parton i to include a ψ that carries a fraction z of the jet momentum.

Fleming [26] pointed out that, since the differential decay rate for the process $Z^0 \rightarrow \psi + \ell^+ \ell^-$ does not vanish in the scaling limit, it must be expressible in the form (11). He found that, at leading order in α and in the scaling limit, it can be written as

$$\begin{aligned} \frac{d\Gamma}{dz}(Z^0 \rightarrow \psi(z) + \ell^+ \ell^-) &= 2 \Gamma(Z^0 \rightarrow \ell^+ \ell^-) D_{\ell \rightarrow \psi}(z, \mu) \\ &+ \frac{d\hat{\Gamma}}{dz}(Z^0 \rightarrow \gamma(z) + \ell^+ \ell^-, \mu) P_{\gamma \rightarrow \psi}, \end{aligned} \quad (12)$$

where $z = 2E/M_Z$. The photon fragmentation probability is $P_{\gamma \rightarrow \psi} = 4\pi\alpha g_\psi^2 = 7 \times 10^{-4}$. The lepton fragmentation function is

$$D_{\ell \rightarrow \psi}(z, \mu) = 2\alpha^2 g_\psi^2 \left[\frac{(z-1)^2 + 1}{z} \log \frac{z\mu^2}{M_\psi^2} - z \right], \quad (13)$$

where μ is an arbitrary factorization scale that separates the scales M_Z and M_ψ . The μ -dependence of the fragmentation function cancels against that of $d\hat{\Gamma}/dz$ in (12). Note that all the effects of QCD, both perturbative and nonperturbative, are absorbed into the factor g_ψ^2 in $P_{\gamma \rightarrow \psi}$ and in (13). In the decay rate for $Z^0 \rightarrow \psi + \ell^+ \ell^-$, the absence of kinematic suppression factors like M_ψ^2/M_Z^2 can be attributed to the fact that the $c\bar{c}$ pair that forms the ψ is produced at a momentum scale of order m_c rather than M_Z . The large momentum scale M_Z only enters in the decay of the Z_0 into the partons ℓ^+ , ℓ^- , and γ .

3.3 Fragmentation Functions in the Color-singlet Model

In 1993, Braaten and Yuan pointed out that the color-singlet model gives predictions for the fragmentation functions for the formation of heavy quarkonium in quark and gluon jets [23]. They calculated the fragmentation functions explicitly for $g \rightarrow \eta_c, \psi$ to leading order in α_s . For example, the fragmentation function for $g \rightarrow \eta_c$ is calculated from the parton process $g \rightarrow c\bar{c} + g$ and is of order α_s^2 :

$$\begin{aligned} D_{g \rightarrow \eta_c}(z, 2m_c) \\ = \frac{\alpha_s^2(2m_c) |R_{\eta_c}(0)|^2}{24\pi m_c^3} [3z - 2z^2 + 2(1-z) \log(1-z)]. \end{aligned} \quad (14)$$

This is the fragmentation function at an initial scale of order $2m_c$. It can be evolved up to a higher scale μ by using the evolution equations (7). The fragmentation function for $g \rightarrow \psi$ is calculated from the parton process $g \rightarrow c\bar{c} + gg$ and is of order α_s^3 . The fragmentation functions for $c \rightarrow \eta_c, \psi$ [27] and $b \rightarrow B_c, B_c^*$ [28] were subsequently calculated by Braaten, Cheung, and Yuan. The fragmentation functions for $b \rightarrow B_c, B_c^*$ had actually been calculated earlier by Chang and Chen [29], although they did not note the universality of these functions.

The fragmentation functions for P-wave quarkonium states have also been calculated in the color-singlet model to leading order in α_s . These fragmentation functions include $g \rightarrow \chi_{cJ}$ [30, 31], $c \rightarrow \chi_{cJ}$, and $b \rightarrow$ P-wave B_c [32, 33, 34]. Some of the fragmentation functions for D-wave states have also been calculated [35, 36]. The color-singlet model

fragmentation functions for $g \rightarrow \chi_{cJ}$ are calculated from the parton process $g \rightarrow c\bar{c} + g$ and are logarithmically infrared-divergent at leading order in α_s . The solution to this problem will be discussed in Section 4. It should be noted that there is a discrepancy between the two calculations of the fragmentation functions for $g \rightarrow \chi_{cJ}$ that has not yet been resolved [30, 31].

The color-singlet model fragmentation functions can be resolved into the contributions from each of the possible values of the helicity h of the charmonium state. The fragmentation functions for $c \rightarrow \psi_L, \psi_T$, where ψ_L and ψ_T represent the longitudinal ($h = 0$) and transverse ($|h| = 1$) polarization components of the ψ , were first calculated by Chen [32] and by Falk et al [37]. The fragmentation functions for $g \rightarrow \chi_{cJ}$ [31, 38], $b \rightarrow B_c^*$ [32, 39], and $b \rightarrow$ P-wave $\bar{b}c$ states [32, 33] have also been resolved into the contributions from the individual helicities.

In the method developed by Braaten and Yuan, the fragmentation functions were extracted by taking the scaling limit of color-singlet model cross sections and expressing them in the factored form demanded by the QCD factorization theorems. This method would be rather cumbersome beyond leading order in α_s . The fragmentation functions can also be calculated directly from the field theoretic definitions [31, 34, 40]. This method provides a significant advantage for calculating fragmentation functions beyond leading order in α_s . Higher order corrections may be particularly important if the leading order calculation gives a soft fragmentation function that is small for z near 1 [41].

4 COLOR-OCTET MECHANISMS

The second major conceptual advance in the recent revolution in heavy quarkonium production is the realization that *color-octet mechanisms* can be important. Contrary to the basic assumption of the color-singlet model, a $c\bar{c}$ pair that is produced in a color-octet state can bind to form charmonium. While a $c\bar{c}$ pair that forms charmonium must ultimately be in a color-singlet state, the short-distance part of the process can involve the production of a color-octet $c\bar{c}$ pair.

In this Section, we first discuss the case of P-wave states, where perturbative consistency requires that a color-octet term be added to the color-singlet model cross section. We then discuss a general factorization formalism for quarkonium that is based on NRQCD. This formalism can be used to factor production cross sections into short-distance parts that can be computed using perturbative QCD and nonperturbative

NRQCD matrix elements. Color-octet mechanisms for S-wave states arise naturally in this framework. Finally, we discuss the implications of this formalism for fragmentation functions.

4.1 *Color-octet Mechanism for P-waves*

The most glaring evidence that the color-singlet model is incomplete comes from the presence of infrared divergences in the production cross sections for P-wave states. There are analogous divergences in the annihilation decay rates for P-wave states, which were discovered by Barbieri and collaborators as early as 1976 [42]. They found that the decay rate for $\chi_{cJ} \rightarrow q\bar{q}g$ depends logarithmically on the minimum energy of the final state gluon. In phenomenological treatments of the annihilation decay rates of P-wave states, the infrared singularity was avoided by imposing an ad hoc infrared cutoff on the gluon energy. This cutoff was sometimes identified with the binding energy of the charmonium state, but without any good justification.

The presence of infrared divergences in the color-singlet model is not a problem if the divergences can be absorbed into the nonperturbative factor in the cross section. For example, there are linear infrared divergences associated with the Coulomb singularity in radiative corrections to S-wave cross sections, but they can be factored into $|R(0)|^2$. Similarly, the radiative corrections for P-waves include a linear infrared divergence that can be factored into $|R'(0)|^2$. However, the structure of the logarithmic infrared divergences for P-waves is such that they cannot be factored into $|R'(0)|^2$.

The solution to this problem was given by Bodwin, Braaten, and Lepage in 1992 [43]. They noted that the infrared divergence in the annihilation rate for $\chi_{cJ} \rightarrow q\bar{q}g$ arises when the final state gluon becomes soft. After radiating the soft gluon from either the c or the \bar{c} in the color-singlet 3P_J bound state, the $c\bar{c}$ pair is in a color-octet 3S_1 state. It then annihilates through the process $c\bar{c} \rightarrow q\bar{q}$. In this region of phase space, the short-distance part of the decay amplitude is therefore the annihilation of a $c\bar{c}$ pair in a color-octet 3S_1 state. Thus, in addition to the conventional term in the color-singlet model, the factorization formula for the decay rate of the χ_{cJ} into light hadrons must include a second term. In the color-singlet model term, the short-distance factor is the annihilation rate of a $c\bar{c}$ pair in a color-singlet 3P_J state and the long-distance nonperturbative factor is proportional to $|R'_{\chi_c}(0)|^2$. In the second term, the short-distance factor is the annihilation rate of a $c\bar{c}$ pair in a color-octet 3S_1 state and the long-distance factor is the probability

for the χ_{cJ} to contain a pointlike $c\bar{c}$ pair in a color-octet 3S_1 state. The color-octet term can be interpreted as a contribution to the decay rate from the $c\bar{c}g$ component of the χ_{cJ} wavefunction.

The solution to the problem of infrared divergences in the production cross sections for the χ_{cJ} states is similar [44]. The infrared divergence arises from the radiation of a soft gluon from the c or \bar{c} that form the color-singlet 3P_J bound state. Before the radiation, the $c\bar{c}$ pair is in a color-octet 3S_1 state. The short-distance part of the process in this region of phase space is the production of a $c\bar{c}$ pair with small relative momentum in a color-octet 3S_1 state. The factorization formula for P-wave cross sections must therefore include two terms:

$$\begin{aligned} d\sigma(\chi_{cJ} + X) &= d\hat{\sigma}(c\bar{c}(\underline{1}, ^3P_J) + X) |R'_{\chi_c}(0)|^2 \\ &\quad + (2J+1) d\hat{\sigma}(c\bar{c}(\underline{8}, ^3S_1) + X) \langle \mathcal{O}_8^{\chi_c} \rangle. \end{aligned} \quad (15)$$

The first term is the conventional term of the color-singlet model. In this term, the short-distance factor is the cross section for producing a $c\bar{c}$ pair with small relative momentum in a color-singlet 3P_J state. The second term in (15) represents a contribution to the cross section from a *color-octet mechanism*. The short-distance factor in this term is the cross section for producing a $c\bar{c}$ pair with small relative momentum in a color-octet 3S_1 state. The long-distance nonperturbative factor $\langle \mathcal{O}_8^{\chi_c} \rangle$ is proportional to the probability for a pointlike $c\bar{c}$ pair in a color-octet 3S_1 state to bind and form a χ_c . The factorization formula (15) has been applied to χ_{cJ} production in B -meson decays [44], in Υ decays [45], and in photoproduction [46].

Both the color-singlet factorization formula (3) for S-waves and the factorization formula (15) for P-waves hold to all orders in perturbation theory in the nonrelativistic limit. This is the limit in which the typical velocity v of the heavy quark in the bound state goes to 0. Unlike the case of S-waves, a color-octet production mechanism is required by perturbative consistency in the case of P-wave states.

This can be understood from the NRQCD factorization approach discussed in Section 4.2. This approach indicates that the color-singlet term for P-waves is suppressed by a factor of v^2 compared to S-waves. It is this suppression of the color-singlet term that makes it necessary to include the color-octet term in the case of P-waves.

4.2 NRQCD Factorization

The simple factorization formulas (3) for S-waves and (15) for P-waves are correct in the nonrelativistic limit, *i.e.* in the limit $v \rightarrow 0$, where v

is the typical relative velocity of the c and \bar{c} in charmonium. Potential model calculations indicate that v^2 is only about $\frac{1}{3}$ for charmonium and about $\frac{1}{10}$ for bottomonium. Thus the neglect of relativistic corrections suppressed by powers of v^2 introduces a systematic theoretical error that may not be negligible. For specific processes, the problem may be worse than suggested by the magnitude of v^2 . If there are other small parameters in the problem, such as $\alpha_s(m_c)$ or kinematical parameters such as m_c^2/p_T^2 , the terms that are of leading order in v^2 may be suppressed by these other parameters. If this suppression is large enough, terms that are subleading in v^2 may actually dominate.

The key to understanding the structure of the relativistic corrections to quarkonium production is to unravel the momentum scales in the problem. If there is a very large momentum scale that is set by the kinematics of the production process, such as M_Z in the case of Z^0 decay or p_T in the case of production at large transverse momentum in $p\bar{p}$ collisions, this scale can be removed from the problem using the factorization theorems of perturbative QCD discussed in Section 3. The next largest momentum scale is the heavy quark mass m_c , which is certainly important in the production of a $c\bar{c}$ pair. The actual formation of the bound state from the $c\bar{c}$ pair involves smaller momentum scales, including the typical momentum $m_c v$ of the heavy quark in quarkonium, its typical kinetic energy $m_c v^2$, and the scale Λ_{QCD} of generic nonperturbative effects in QCD. In charmonium and bottomonium, the mass m_Q is large enough that effects associated with this scale should be calculable using perturbation theory in the running coupling constant $\alpha_s(m_Q)$. Nonperturbative effects become important at smaller momentum scales.

A powerful tool for isolating the effects of the scale m_Q is *nonrelativistic QCD* (NRQCD), an effective field theory developed by Caswell and Lepage [47]. NRQCD is a formulation of QCD in which the heavy quark and antiquark are treated nonrelativistically. They are described by a Schroedinger field theory with separate 2-component spinor fields, rather than a single 4-component Dirac field. The gluons and light quarks are described by the relativistic lagrangian for ordinary QCD.

NRQCD can accurately describe the physics of heavy quarkonium at energies that are much less than m_Q above the rest mass. This follows from the fact that virtual states with energies of order m_Q or larger have lifetimes of order $1/m_Q$ or smaller. In this time, quanta can propagate over distances that are at most of order $1/m_Q$. Since this distance is much smaller than the size of the quarkonium bound state, the effects of these virtual states can be taken into account through local interaction terms in the NRQCD lagrangian.

The inverse of the mean radius of

a quarkonium state is a dynamically generated momentum scale that is much smaller than its rest mass. The ratio of these momentum scales defines a small parameter v which can be identified with the typical velocity of the heavy quark in the quarkonium state. This small parameter v , which is defined nonperturbatively, can be exploited to organize calculations of heavy quarkonium observables into expansions in powers of v^2 . NRQCD is equivalent to full QCD in the sense that the parameters in the NRQCD lagrangian can be tuned so that its predictions agree with those of full QCD to any desired order in v^2 [48].

Bodwin, Braaten, and Lepage [49] have recently developed a theoretical framework for inclusive quarkonium production that allows relativistic corrections to be included to any desired order in v^2 . This framework provides a factorization formula for inclusive cross sections in which all effects of the scale m_c are separated from the effects of lower momentum scales, including $m_c v$. The derivation of this factorization formula involves two steps. This first step is the topological factorization of diagrams that contribute to the cross section. This step is similar to the derivations of the factorization theorems of perturbative QCD that were discussed in section 3.1. For simplicity, we consider the case of charmonium production in e^+e^- annihilation at a center-of-mass energy \sqrt{s} that is significantly larger than $2m_c$. We define a *scaling limit* by $\sqrt{s}, m_c \rightarrow \infty$ with \sqrt{s}/m_c fixed. A diagram that contributes to the cross section in this limit can be separated into a hard-scattering subdiagram that produces hard partons and a $c\bar{c}$ pair with small relative momentum, a jet-like subdiagram for each of the hard partons, a subdiagram that involves the c and \bar{c} , and a soft part. The soft part includes soft gluons that can couple to the jet-like subdiagrams and to the $c\bar{c}$ subdiagram. After summing over all possible connections of the soft gluons, the effects of the soft gluons that are connected to the jet-like subdiagrams cancel, leaving a factored form for this contribution to the cross section. The derivation of topological factorization requires the hard partons to all have large momentum relative to the $c\bar{c}$ pair. This can presumably be generalized to processes in which there are hadrons in the initial state, provided that the charmonium is produced with transverse momentum large compared to Λ_{QCD} .

After the topological factorization of the diagram, the hard-scattering amplitude contains all effects of the scale m_c , but it also depends on the relative momentum \mathbf{p} of the $c\bar{c}$ pair. To complete the derivation of the factorization formula, the dependence on \mathbf{p} must be removed from the hard-scattering amplitude, so that all effects of the scale $m_c v$ reside

in the $c\bar{c}$ subdiagram. This could be accomplished simply by Taylor-expanding the hard-scattering amplitude in powers of \mathbf{p} , if it were not for the fact that this generates ultraviolet divergences in the $c\bar{c}$ subdiagram. This is where the power of NRQCD becomes evident. This effective field theory can be used to systematically unravel the scales m_c and $m_c v$, so that all effects of the scale $m_c v$ are contained in the $c\bar{c}$ subdiagram. Essentially, the $c\bar{c}$ subdiagram is defined to include all the parts of the diagram that are reproduced by NRQCD. Those parts involving the $c\bar{c}$ pair that are not reproduced by NRQCD must involve the scale m_c , and therefore can be included in the hard-scattering subdiagram. The result is a factorization formula for the inclusive cross section for producing a quarkonium state H that holds to all orders in α_s . It has the form

$$d\sigma(H + X) = \sum_n d\hat{\sigma}(c\bar{c}(n) + X) \langle \mathcal{O}_n^H \rangle, \quad (16)$$

where $d\hat{\sigma}$ is the inclusive cross section for producing a $c\bar{c}$ pair in a color and angular-momentum state labeled by n and having vanishing relative momentum. The parton cross sections $d\hat{\sigma}$ involve only momenta of order m_c or larger, and therefore they can be calculated as perturbation expansions in $\alpha_s(m_c)$. Since $d\hat{\sigma}$ is insensitive to relative momenta that are much smaller than m_c , it describes the production of a $c\bar{c}$ pair with separation less than or of order $1/m_c$. This separation is essentially pointlike on the scale of a charmonium wavefunction, which is of order $1/(m_c v)$. Thus the nonperturbative long-distance factor $\langle \mathcal{O}_n^H \rangle$ is proportional to the probability for a pointlike $c\bar{c}$ pair in the state n to form the bound state H . Note that the only dependence on the quarkonium state H in (16) resides in the factor $\langle \mathcal{O}_n^H \rangle$.

The factorization formula (16) holds only for *inclusive* cross sections. The matrix elements $\langle \mathcal{O}_n^H \rangle$ are proportional to the probabilities for the formation of the charmonium state H , plus light hadrons whose total energy in the H rest frame is of order $m_c v^2$. Summation over these light hadronic states is essential for the separation of short-distance and long-distance effects. The methods required to derive the factorization formula break down for exclusive cross sections. The derivation also implies that factorization does not hold at the amplitude level, contrary to the basic assumption of the color-singlet model.

The factors $\langle \mathcal{O}_n^H \rangle$ in (16) can be expressed as vacuum matrix elements of 4-quark operators in NRQCD [49]. The operator \mathcal{O}_n^H creates a pointlike $c\bar{c}$ pair in the state n , projects onto states that in the asymptotic future include the quarkonium state H , and finally annihilates the $c\bar{c}$ pair at the creation point, again in the state n . The matrix elements

that play the most important roles in quarkonium production were denoted $\langle \mathcal{O}_1^H(^{2S+1}L_J) \rangle$ and $\langle \mathcal{O}_8^H(^{2S+1}L_J) \rangle$ in Ref. [49]. The operator $\mathcal{O}_n(^{2S+1}L_J)$ creates and annihilates a $c\bar{c}$ pair in the state $c\bar{c}(\underline{n}, ^{2S+1}L_J)$. If the dominant Fock state of the meson H is $|c\bar{c}(\underline{1}, ^{2S+1}L_J)\rangle$, the matrix element $\langle \mathcal{O}_1^H(^{2S+1}L_J) \rangle$ can be related to the radial wavefunction. For example, up to corrections of relative order v^4 , we have

$$\langle \mathcal{O}_1^\psi(^3S_1) \rangle = \frac{9}{2\pi} |R_\psi(0)|^2, \quad (17)$$

$$\langle \mathcal{O}_1^{\chi_{cJ}}(^3P_J) \rangle = (2J+1) \frac{9}{2\pi} |R'_{\chi_c}(0)|^2. \quad (18)$$

If we keep only these terms in the factorization formula (16), we recover the formulas (3) and (4) of the color-singlet model. NRQCD has approximate heavy-quark spin symmetry, and this implies relations between color-octet matrix elements. For example, up to corrections of relative order v^2 , we have

$$\langle \mathcal{O}_8^{\chi_{cJ}}(^3S_1) \rangle = (2J+1) \langle \mathcal{O}_8^{\chi_{c0}}(^3S_0) \rangle. \quad (19)$$

Identifying $\langle \mathcal{O}_8^{\chi_{c0}}(^3S_1) \rangle = \langle \mathcal{O}_8^{\chi_c} \rangle$, one finds that the corresponding term in the factorization formula (16) reproduces the color-octet term in the nonrelativistic factorization formula (15) for P-wave production.

The factorization formula (16) is not particularly useful in its most general form, since it involves infinitely many nonperturbative factors $\langle \mathcal{O}_n^H \rangle$. However, one can deduce from NRQCD how the various matrix elements scale with v for any particular quarkonium state H . For example, the scaling of $\langle \mathcal{O}_n^H(^{2S+1}L_J) \rangle$ with v is determined by the number of electric dipole and magnetic dipole transitions that are required to go from the dominant Fock state of the meson H to a state of the form $|c\bar{c}(\underline{n}, ^{2S+1}L_J) + \text{gluons}\rangle$. The matrix element scales as v^{3+2L} , multiplied by v^2 for each electric dipole transition and v^4 for each magnetic dipole transition.

The relative importance of the various terms in the factorization formula (16) is determined by the order in v of the matrix element $\langle \mathcal{O}_n^H \rangle$ and by the order in $\alpha_s(m_c)$ of the parton cross sections $d\hat{\sigma}(c\bar{c}(n) + X)$. The NRQCD factorization framework acquires predictive power when the expansion (16) is truncated to some low order in v^2 . If we truncate (16) to lowest nontrivial order in v , regardless of the order in α_s , we recover the factorization formulas (3) for S-waves and (15) for P-waves. However, if the cross sections $d\hat{\sigma}$ for these terms are suppressed, other terms in the factorization formula may be important. This observation is the basis for a suggestion by Braaten and Fleming [50] that *color-octet*

mechanisms may also play an important role in S-wave production. The leading color-octet matrix elements for the ψ are $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$, $\langle \mathcal{O}_8^\psi(^3S_1) \rangle$, and $\langle \mathcal{O}_8^\psi(^3P_J) \rangle$, all of which are suppressed by v^4 relative to the leading color-singlet matrix element $\langle \mathcal{O}_1^\psi(^3S_1) \rangle$.

If the parton cross sections $d\hat{\sigma}$ multiplying the color-octet matrix elements have the same magnitudes as the parton cross section multiplying $\langle \mathcal{O}_1^\psi(^3S_1) \rangle$, then the corrections from the color-octet terms should be small. However, if the parton cross section multiplying the color-singlet matrix element is suppressed by powers of $\alpha_s(m_c)$ or by small kinematical parameters such as m_c^2/p_T^2 , the color-octet terms could dominate. An example is the gluon fragmentation function for producing a ψ , which is discussed in Section 4.3.

The implications of color-octet mechanisms for the production of quarkonium in high energy colliders are discussed in Section 5. There have also been many recent investigations of color-octet production mechanisms in lower energy experiments, including fixed target πN and pN collisions [51], B -meson decays [52], e^+e^- annihilation [53], and photoproduction [54, 55]. These applications lie outside the scope of this review.

4.3 Fragmentation Functions for Heavy Quarkonium

The NRQCD factorization formula (16) implies that fragmentation functions for charmonium have the general form

$$D_{i \rightarrow H}(z, \mu) = \sum_n d_{i \rightarrow n}(z, \mu) \langle \mathcal{O}_n^H \rangle, \quad (20)$$

where $d_{i \rightarrow n}(z, \mu)$ gives the probability for the parton i to form a jet that includes a $c\bar{c}$ pair in the state labeled by n , and $\langle \mathcal{O}_n^H \rangle$ is proportional to the probability for a pointlike $c\bar{c}$ pair in the state n to bind to form a charmonium state H . The coefficient $d_{i \rightarrow n}(z, 2m_c)$ at the initial scale $\mu = 2m_c$ involves only momenta of order m_c , and can therefore be calculated as a perturbation expansion in the running coupling constant $\alpha_s(2m_c)$. The fragmentation function can be evolved up to higher scales μ by using the evolution equations (7). Note that the only dependence on the quarkonium state H on the right side of (20) resides in the matrix elements.

If the dominant Fock state for the meson H is $|c\bar{c}(1, ^{2S+1}L_J)\rangle$, the fragmentation function in the color-singlet model is obtained by keeping only the term in (20) that involves $\langle \mathcal{O}_1^H(^{2S+1}L_J) \rangle$. In the case of

gluon fragmentation into χ_{cJ} , the coefficient of $\langle \mathcal{O}_1^{\chi_{cJ}}(^3P_J) \rangle$ is logarithmically infrared divergent at leading order in α_s . The corresponding parton process is $g \rightarrow c\bar{c}(\underline{1}, ^3P_J) + g$, and the divergence arises when the final state gluon becomes soft. In this region of phase space, the short-distance part of the fragmentation process is $g \rightarrow c\bar{c}(\underline{8}, ^3S_1)$, and the divergence from the radiation of the soft gluon should be absorbed into the matrix element $\langle \mathcal{O}_8^{\chi_{cJ}}(^3S_1) \rangle$. The resulting expression for the fragmentation function at leading order in α_s is [30]

$$D_{g \rightarrow \chi_{cJ}}(z, 2m_c) = \frac{\alpha_s^2(2m_c)}{m_c^5} d_J(z) \langle \mathcal{O}_1^{\chi_{cJ}}(^3P_J) \rangle + \frac{\pi\alpha_s(2m_c)}{24m_c^3} \delta(1-z) \langle \mathcal{O}_8^{\chi_{cJ}}(^3S_1) \rangle, \quad (21)$$

where $d_J(z)$ is a dimensionless function of z . Since the matrix elements are the same order in v and the color-octet term is lower order in α_s , it can be expected to dominate.

The process $g \rightarrow c\bar{c}(\underline{8}, ^3S_1)$ gives a term of order α_s in the gluon fragmentation function for any quarkonium state H :

$$D_{g \rightarrow H}(z, 2m_c) \approx \frac{\pi\alpha_s(2m_c)}{24m_c^3} \delta(1-z) \langle \mathcal{O}_8^H(^3S_1) \rangle. \quad (22)$$

The delta function in (22) should be interpreted as a distribution in z that is peaked near $z = 1$ with a width of order v^2 . All other terms in the fragmentation function have coefficients $d_{g \rightarrow n}(z)$ of order α_s^2 or higher, or else have matrix elements that are higher order in v^2 than $\langle \mathcal{O}_8^H(^3S_1) \rangle$. The importance of the term (22) in the fragmentation function depends on the charmonium state H . Surprisingly, it may be the most important term in the fragmentation functions for the S-wave states ψ and ψ' . Although the matrix element $\langle \mathcal{O}_8^\psi(^3S_1) \rangle$ is suppressed by v^4 relative to $\langle \mathcal{O}_1^\psi(^3S_1) \rangle$, the leading term in the coefficient of the color-singlet matrix element comes from the parton process $g \rightarrow c\bar{c}(\underline{1}, ^3S_1) + gg$ and is therefore of order α_s^3 . The suppression of the color-singlet term by α_s^2 may be more effective than the suppression of the color-octet term by v^4 . The relative importance of the two terms can only be assessed after determining the value of the color-octet matrix element. An estimate for this matrix element will be obtained from Tevatron data in Section 5.1.

5 COLLIDER APPLICATIONS

Our understanding of heavy-quarkonium production in high energy colliders has been completely revolutionized by the theoretical developments described in Sections 3 and 4. It is clear that fragmentation contributions can dominate at large transverse momentum, even if they are higher order in α_s . In addition, the formation of quarkonium from color-octet $Q\bar{Q}$ pairs can in some cases dominate over the color-singlet contributions. In this Section, we discuss several applications to collider physics in which these developments play an important role. These applications are the production of prompt charmonium at the Tevatron, the production of bottomonium and prompt charmonium at LEP, and the production of the B_c in high energy colliders.

5.1 *Prompt Charmonium at Large p_T in $p\bar{p}$ Collisions*

Much of the impetus for the recent theoretical developments in heavy quarkonium production has come from Tevatron data on prompt charmonium production at large p_T . As discussed in Section 2.3, the experimentally measured rate at large p_T is orders of magnitude greater than the predictions of the color-singlet model at leading order in α_s . However, the Tevatron results can be explained by a combination of the fragmentation and color-octet mechanisms discussed in Sections 3 and 4.

In 1993, Braaten and Yuan [23] pointed out that fragmentation should be the most important charmonium production mechanism at sufficiently large p_T . Doncheski, Fleming, and Mangano [56] carried out the first explicit calculation of the fragmentation contribution to prompt charmonium production. They used the gluon and charm quark fragmentation functions from the color-singlet model to calculate the fragmentation contributions to direct ψ and direct ψ' production at large p_T at the Tevatron. They found that, although fragmentation does indeed dominate over the leading-order color-singlet contribution for p_T greater than about 7 GeV, the predictions still fell more than an order of magnitude below the CDF data for inclusive ψ and ψ' production.

In subsequent calculations of prompt ψ production [57], contributions from fragmentation into direct χ_c , followed by the radiative decay of χ_{cJ} into ψ , were included. These contributions are roughly an order of magnitude larger than the direct ψ contributions, and are dominated by the color-octet term in the gluon fragmentation function for χ_{cJ} . The value of the NRQCD matrix element $\langle \mathcal{O}_8^{\chi_{c0}}(^3S_1) \rangle$ that was used in this

calculation was estimated from data on B -meson decays [44]. The inclusion of the contribution from gluon fragmentation into χ_{cJ} brings the theoretical prediction for inclusive ψ production to within a factor of 3 of the CDF data [16]. Because of the many theoretical uncertainties that enter into the calculation, this factor of 3 discrepancy was considered acceptable at the time. However, in the case of prompt ψ' production, the theoretical prediction remained about a factor of 30 below the data. This dramatic discrepancy became known as the “CDF ψ' anomaly”.

The main difference between the prompt production of ψ and ψ' is that the ψ signal is fed by direct χ_{cJ} 's, while the ψ' signal is not fed by any known charmonium states. The ψ' anomaly could be explained if there exist undiscovered charmonium states with sufficiently large branching fractions into ψ' . Among the possibilities that have been considered are D-wave states, higher P-wave states, and “hybrid” states whose dominant Fock state is $c\bar{c}g$ [38, 58]. The main difficulty with these proposals is explaining why these states should have the large branching fractions into ψ' that would be required to explain the data.

An alternative solution to the ψ' anomaly, based on the NRQCD factorization formalism, was proposed by Braaten and Fleming [50]. They suggested that the dominant contribution to the production of ψ' with large p_T comes from the color-octet term (22) in the gluon fragmentation function for ψ' . This term represents the fragmentation of a gluon into a $c\bar{c}$ pair in a color-octet 3S_1 state at short distances, followed by the formation of the ψ' from the $c\bar{c}$ pair through nonperturbative QCD interactions. The probability for the formation of the ψ' is proportional to the NRQCD matrix element $\langle \mathcal{O}_8^{\psi'}(^3S_1) \rangle$. While the normalization of this term in the cross section for ψ' production depends on the undetermined matrix element $\langle \mathcal{O}_8^{\psi'}(^3S_1) \rangle$, the p_T -dependence of this contribution is predicted and found to be in good agreement with the CDF data [16]. By fitting to CDF data, one finds that the value of the matrix element is $\langle \mathcal{O}_8^{\psi'}(^3S_1) \rangle \approx 0.0042 \text{ GeV}^3$. This value is consistent with the NRQCD prediction that it should be suppressed by v^4 with respect to the corresponding color-singlet matrix element $\langle \mathcal{O}_1^{\psi'}(^3S_1) \rangle \approx 0.573 \text{ GeV}^3$. Thus this solution to the ψ' anomaly is at least plausible.

This proposal was given further support by subsequent CDF data in which the prompt ψ 's were separated into those that come from χ_c decay and those that do not [17]. The observed rate for prompt ψ 's not from χ_c decay was about a factor of 30 larger than the theoretical predictions for direct ψ from fragmentation in the color-singlet model.

Cacciari, Greco, Mangano, and Petrelli showed that this data could be explained by color-octet gluon fragmentation [59, 60]. As in the case of ψ' , the p_T -dependence predicted by color-octet fragmentation is in good agreement with the data. The value of the color-octet matrix element that is required to fit the data is $\langle \mathcal{O}_8^\psi(^3S_1) \rangle \approx 0.014 \text{ GeV}^3$. This is small compared to the corresponding color-singlet matrix element $\langle \mathcal{O}_1^\psi(^3S_1) \rangle \approx 1.00 \text{ GeV}^3$, consistent with the suppression by v^4 predicted by NRQCD.

The simple picture that seems to emerge from the CDF data is that the dominant mechanism for the production of ψ , ψ' , or χ_{cJ} at large p_T is the perturbative fragmentation of a gluon into a $c\bar{c}$ pair in a color-octet 3S_1 state, followed by the nonperturbative formation of charmonium from the $c\bar{c}$ pair. The differential cross section therefore reduces to that for producing a gluon convoluted with a fragmentation function:

$$\begin{aligned} d\sigma(p\bar{p} \rightarrow H(p_T) + X) &= \sum_{jk} \int_0^1 dx_1 f_{j/p}(x_1) \int_0^1 dx_2 f_{k/\bar{p}}(x_2) \\ &\times \int_0^1 dz d\hat{\sigma}(jk \rightarrow g(p_T/z) + X, \mu_{\text{frag}}) D_{g \rightarrow H}(z, \mu_{\text{frag}}). \end{aligned} \quad (23)$$

The dominant term in the gluon fragmentation function is assumed to be the term proportional to $\langle \mathcal{O}_8^H(^3S_1) \rangle$. It is given by (22) at the scale $\mu_{\text{frag}} = 2m_c$, but it should be evolved up to the scale $\mu_{\text{frag}} = p_T/z$ using the evolution equations (7). Of course, a thorough analysis of charmonium production at large p_T must include additional terms in the gluon fragmentation function that are of order α_s^2 and higher. It must also include the contributions from the fragmentation of other partons, such as the charm quark and light quarks.

In Figures 1, 2, and 3, the predictions of this simple picture of charmonium production are compared with the most recent CDF data on prompt ψ not from χ_c decay [17], prompt ψ from χ_{c1} and χ_{c2} decay [17], and prompt ψ' [16], respectively. The predictions of the color-singlet model at leading order in α_s are shown as dashed lines and they fall orders of magnitude below the data. The dotted lines in Figures 1 and 2 are the fragmentation contributions calculated in the color-singlet model. While they increase the theoretical predictions by more than an order of magnitude at the largest values of p_T , they still fall about a factor of 30 below the experimental measurements. The solid curves in Figures 1, 2, and 3 are the contributions from color-octet fragmentation, with the color-octet matrix elements $\langle \mathcal{O}_8^H(^3S_1) \rangle$ adjusted to fit the data. The solid curves differ only in their normalizations. The normalizations

in Figures 1, 2, and 3 are proportional to $\langle \mathcal{O}_8^\psi(^3S_1) \rangle + 0.57\langle \mathcal{O}_8^{\psi'}(^3S_1) \rangle$, $0.27\langle \mathcal{O}_8^{\chi_{c1}}(^3S_1) \rangle + 0.14\langle \mathcal{O}_8^{\chi_{c2}}(^3S_1) \rangle = 1.51\langle \mathcal{O}_8^{\chi_{c0}}(^3S_1) \rangle$, and $\langle \mathcal{O}_8^{\psi'}(^3S_1) \rangle$, respectively. Their shapes agree reasonably well with the data. The values of the matrix elements for ψ and ψ' are given above. The value $\langle \mathcal{O}_8^{\chi_{c0}}(^3S_1) \rangle = 0.0076 \text{ GeV}^3$ obtained for the χ_c matrix element is significantly larger than the most recent value obtained from B -meson decay [50]. In calculating these curves, we imposed a pseudorapidity cut of $|\eta| < 0.6$ on the charmonium state and we used the MRSD0 parton distribution set. The scales μ_R , μ_F , and μ_{frag} were all chosen to be equal to the transverse momentum p_T/z of the fragmenting gluon.

While the fragmentation contribution must dominate at sufficiently large p_T , the contributions to the cross section that are suppressed by factors of m_c^2/p_T^2 may be important at the values of p_T that are experimentally accessible. These contributions have been included in a recent calculation by Cho and Leibovich [61]. They calculated the cross sections for all $2 \rightarrow 3$ parton processes of the form $ij \rightarrow c\bar{c} + k$ that produce $c\bar{c}$ pairs in color-octet states with angular momentum quantum numbers 3S_1 , 1S_0 , and 3P_J . They used these to calculate the cross section for direct ψ and ψ' production, including all color-octet contributions that are suppressed only by v^4 . By fitting the p_T distributions for all p_T greater than 5 GeV, they extracted values for the matrix elements $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$ and $\langle \mathcal{O}_8^\psi(^3P_J) \rangle$, as well as $\langle \mathcal{O}_8^\psi(^3S_1) \rangle$. The best fit requires a value for $\langle \mathcal{O}_8^\psi(^3S_1) \rangle$ that is significantly smaller than that obtained above. The terms involving $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$ and $\langle \mathcal{O}_8^\psi(^3P_J) \rangle$ dominate in the range $5 \text{ GeV} < p_T < 10 \text{ GeV}$. At $p_T > 10 \text{ GeV}$, the term involving $\langle \mathcal{O}_8^\psi(^3S_1) \rangle$ dominates and it asymptotically approaches the contribution from the leading color-octet term in the gluon fragmentation function. The difference between the full calculation of the differential cross section and the fragmentation approximation is less than 20% at $p_T = 10 \text{ GeV}$.

The greatest weakness of the proposal to explain the CDF data by color-octet production mechanisms is that the color-octet matrix elements are free parameters that can be adjusted to fit the data. The predictive power in this proposal lies in the fact that the NRQCD matrix elements are universal, and will also enter into other charmonium production processes. The most convincing proof that the color-octet proposal is correct would be to show that the values of the matrix elements determined from the CDF data are required to explain charmonium production in other high energy processes. One possibility is Z^0 decay, which will be discussed in Section 5.2.

The color-octet fragmentation mechanism also gives other predic-

tions that can be tested experimentally. The most dramatic prediction is that direct ψ 's and ψ 's at large p_T will have a large spin alignment. Cho and Wise [62] pointed out that ψ 's and ψ 's produced by color-octet gluon fragmentation tend to inherit the transverse polarization of the nearly on-shell gluon. The process $g \rightarrow c\bar{c}(\underline{8}, {}^3S_1)$ produces a $c\bar{c}$ pair whose total spin is transversely polarized. The approximate heavy-quark spin symmetry of NRQCD implies that the spin state of the $c\bar{c}$ pair will be affected very little by the nonperturbative QCD effects that are involved in binding the $c\bar{c}$ pair into a ψ . At leading order in α_s , the predicted polarization is 100% transverse. Beneke and Rothstein analyzed the radiative corrections and concluded that they can decrease the polarization by only about 10% [63]. Cho and Leibovich [61] have shown that the dominant corrections to the spin alignment at Tevatron energies come from terms in the cross section that involve $\langle \mathcal{O}_8^\psi({}^1S_0) \rangle$ and $\langle \mathcal{O}_8^\psi({}^3P_J) \rangle$ and fall as m_c^2/p_T^2 . These corrections remain small at the largest values of p_T measured at the Tevatron. A large transverse spin alignment is therefore a robust signature for the dominance by color-octet fragmentation of the production of direct ψ 's and ψ 's at high p_T .

There are other predictions of the color-octet fragmentation mechanism that can be tested at the Tevatron. As pointed out by Barger, Fleming, and Phillips, the production of two quarkonium states with large p_T will be dominated by the production of two high- p_T gluons which both fragment into quarkonium [64]. Taking into account the color-octet mechanism, they found that $\psi\psi$ events at large p_T should be detectable at the Tevatron. The color-octet fragmentation mechanism also gives predictions for correlations between charmonium at large p_T and the other jets produced by the $p\bar{p}$ collision, but these have not yet been explored.

The differential cross sections for the production of bottomonium states at the Tevatron have also been calculated by Cho and Leibovich [60, 61]. The predictions of the color-singlet model fall more than an order of magnitude below the CDF data for $p_T > 5$ GeV [65]. The production rates can, however, be explained by including color-octet terms in the cross section for the states $\Upsilon(nS)$ and $\chi_{bJ}(n)$, $n = 1, 2, 3$. The values of the color-octet matrix elements that are required to fit the CDF data are consistent with expectations from NRQCD. The calculations of Cho and Leibovich show that fragmentation is not important in bottomonium production at the Tevatron. The error from neglecting contributions that fall as m_c^2/p_T^2 is greater than 100% even if p_T is as large as 20 GeV.

Color-singlet and color-octet fragmentation processes in inelastic ψ photoproduction at the HERA ep collider have also been investigated [55, 66]. The dominant contributions for values of p_T accessible to experimental studies appear to be color-singlet charm quark fragmentation and color-octet processes that are suppressed by m_c^2/p_T^2 . In the color-octet contributions the $c\bar{c}$ pair is created at short distances in either a 1S_0 or 3P_J state. Because the numerical values of the color-octet matrix elements $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$ and $\langle \mathcal{O}_8^\psi(^3P_J) \rangle$ are not measured very precisely it is not possible to determine which process is the dominant one at HERA. However at asymptotic values of p_T the fragmentation contribution will be the most important contribution. Thus for large p_T data from HERA can be used to study a fragmentation mechanism that is not so easily probed at the Tevatron and at LHC.

5.2 Charmonium and Bottomonium in Z^0 Decay

The decay of the Z^0 provides a laboratory for the study of heavy quarkonium with large transverse momentum that is complementary to $p\bar{p}$ collisions. The color-singlet model predicts that the production rates for prompt charmonium and for bottomonium are so small that they are unlikely to be observed at LEP. However, the dramatic failure of the color-singlet model in $p\bar{p}$ collisions suggests that its predictions for Z^0 decay should also be reexamined.

The predictions of the color-singlet model for charmonium production in Z^0 decay have been studied thoroughly [5, 7, 67, 68]. The largest production process for ψ in the color-singlet model is $Z^0 \rightarrow \psi c\bar{c}$. The rate for this process was first calculated in 1990 by Barger, Cheung, and Keung [67], who found it to be surprisingly large. It is two orders of magnitude larger than the rate for $Z^0 \rightarrow \psi gg$ [7], despite the fact that both processes are of the same order in α_s . This was explained in 1993 by Braaten, Cheung, and Yuan, who pointed out that the reason $Z^0 \rightarrow \psi c\bar{c}$ is so large is that this process has a fragmentation contribution that is not suppressed by m_c^2/M_Z^2 [27]. They showed that the dominant terms can be factored into the decay rate for $Z^0 \rightarrow c\bar{c}$ and fragmentation functions for the processes $c \rightarrow \psi c$ and $\bar{c} \rightarrow \psi\bar{c}$:

$$\frac{d\Gamma}{dz_\psi}(Z^0 \rightarrow \psi(z_\psi) + X) \approx 2 \Gamma(Z^0 \rightarrow c\bar{c}) D_{c \rightarrow \psi}(z_\psi), \quad (24)$$

where $z_\psi = 2E_\psi/M_Z$. Since the momentum scale for the fragmentation process is of order m_c , the rate is proportional to $\alpha_s^2(m_c)$ instead

of $\alpha_s^2(M_Z)$. The resulting prediction for the branching fraction for direct ψ production in Z^0 decay is 2.9×10^{-5} . The experimental study of prompt charmonium in Z^0 decay is complicated by the large background from the decay of b -hadrons, which must be removed using vertex detectors. There are preliminary results from LEP [69] that indicate that the branching fraction for prompt ψ production is around 10^{-4} , well above the predictions of the color-singlet model.

Another fragmentation process in the color-singlet model was considered by Hagiwara, Martin, and Stirling in 1991 [68]. They calculated the rates for the process $Z^0 \rightarrow q\bar{q}g^*$, with charmonium being produced by the virtual gluon through the processes $g^* \rightarrow \psi gg$ and $g^* \rightarrow \chi_{cJ} g$. The rate is at least an order of magnitude smaller than the charm fragmentation process described above, but the experimental backgrounds are less severe. The DELPHI collaboration [69] has recently reported 4.1×10^{-4} as a limit on the branching fraction for $Z^0 \rightarrow \psi X$ from this process.

The color-octet mechanism that was introduced to explain prompt charmonium production at the Tevatron also offers new possibilities for charmonium production at the Z^0 resonance. Cheung, Keung, and Yuan [70] and Cho [71] have studied prompt charmonium production at LEP via the color-octet mechanism. The color-octet process that is leading order in α_s involves the short-distance decay $Z^0 \rightarrow c\bar{c}(\underline{8}, {}^3S_1) + g$, but it has a negligible branching fraction because it is suppressed by a short-distance factor of m_c^2/M_Z^2 . The dominant color-octet process is of order α_s^2 and involves the short-distance decay $Z^0 \rightarrow c\bar{c}(\underline{8}, {}^3S_1) + q\bar{q}$, where q can be one of the light quarks u, d , or s or one of the heavy quarks c or b . Apart from different coupling constants, color factors, and long-distance matrix elements, this process is identical to the electromagnetic production of ψ studied by Fleming [26] and discussed in Section 3.2. At leading order in α_s , the perturbative QCD factorization formula for this process has the form

$$\begin{aligned} \frac{d\Gamma}{dz_\psi}(Z^0 \rightarrow \psi(z_\psi) + X) &\approx 2 \sum_q \Gamma(Z^0 \rightarrow q\bar{q}) D_{q \rightarrow \psi}(z_\psi, \mu) \\ &+ \sum_q \int_{z_\psi}^1 dy \frac{d\hat{\Gamma}}{dz_g}(Z^0 \rightarrow g(z_\psi/y)q\bar{q}, \mu) D_{g \rightarrow \psi}(y), \end{aligned} \quad (25)$$

where $z_\psi = 2E_\psi/M_Z$ and $z_g = z_\psi/y$. The fragmentation function $D_{g \rightarrow \psi}(y)$ for the formation of a ψ in a gluon jet through the color-octet mechanism is given in (22), except that it must be evolved up to the scale E_ψ/y set by the gluon energy. The fragmentation function $D_{q \rightarrow \psi}(z)$ for

the formation of a ψ in a light quark jet can be obtained from (13) by replacing $\alpha^2 g_\psi^2$ by $\alpha_s^2 \langle \mathcal{O}_1^\psi(^3S_1) \rangle / (72m_c^3)$.

In Figure 4, we compare the ψ energy distribution predicted by the color-singlet model with the distribution from the most important color-octet process $Z^0 \rightarrow c\bar{c}(\underline{8}, ^3S_1) + q\bar{q}$. The most striking feature in Figure 4 is that the energy distribution of the color-octet process dominates over that of the color-singlet model for all values of $z_\psi = 2E_\psi/M_Z$. There is also a dramatic difference between the shapes of the two distributions. The energy distribution from the color-singlet process is rather hard because of the nature of heavy quark fragmentation. The distribution for the color-octet process is very soft and has a pronounced peak near $z_\psi = 0.1$. The prediction for the branching ratio for $Z^0 \rightarrow \psi + X$ from the color-octet process is 1.4×10^{-4} . This is consistent with the DELPHI bound for the color-singlet process $Z^0 \rightarrow \psi g g q \bar{q}$, which produces ψ 's with a similar event topology.

The color-octet fragmentation process is also important in the production of ψ' and χ_c . The energy distributions are predicted to be similar to that for the ψ shown in figure 4. The production rates are predicted to be smaller by about a factor of 3 for the ψ' and larger by about a factor of 5 for the three spin states of the χ_{cJ} combined.

Just as in the case of $p\bar{p}$ annihilation, the color-octet fragmentation mechanism makes definite predictions for the spin alignment of the ψ in Z^0 decay. The color-octet process $Z^0 \rightarrow c\bar{c}(\underline{8}, ^3S_1) + q\bar{q}$ involves contributions from both gluon fragmentation and light quark fragmentation. At leading order in α_s , gluon fragmentation only produces ψ 's that are transversely polarized. However, a light quark can fragment into a longitudinally polarized ψ even at leading order in α_s . This leads to significant degradation of the spin alignment of the ψ . The spin alignment varies significantly with z , but the average transverse polarization may be as small as 75% [72].

Bottomonium states can also be produced in Z^0 decay. These states are simpler to study experimentally, since there is no background to bottomonium production from non-prompt production mechanisms analogous to b -hadrons decay into charmonium. A signal from the $J^{PC} = 1^{--}$ states $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ has in fact been observed at LEP. The preliminary result from the OPAL detector is that the branching fraction summed over the three states is [73]

$$\sum_{n=1}^3 \text{Br}(Z^0 \rightarrow \Upsilon(nS) + X) = (1.2_{-0.6}^{+0.9} \pm 0.2) \times 10^{-4} . \quad (26)$$

The direct production mechanisms for $\Upsilon(nS)$ in Z^0 decay are essentially identical to those for prompt ψ . The dominant color-singlet process is the decay $Z^0 \rightarrow \Upsilon(nS) + b\bar{b}$. This process has a fragmentation contribution which corresponds to the decay $Z^0 \rightarrow b\bar{b}$ followed by the formation of $\Upsilon(nS)$ through the fragmentation of the b or \bar{b} . The prediction for the sum of branching fraction given in (26) is 1.6×10^{-5} , which is about an order of magnitude below the OPAL result. Color-octet mechanisms for the production of $\Upsilon(nS)$ and $\chi_{bJ}(n)$ have been studied by Cho [71]. The dominant mechanism is the formation of bottomonium through the short-distance process $Z^0 \rightarrow b\bar{b}(\underline{8}, {}^3S_1) + q\bar{q}$, $q = u, d, s$, which involves both gluon fragmentation and light quark fragmentation. Using values for the color-octet matrix elements $\langle \mathcal{O}_8^H({}^3S_1) \rangle$ for the bottomonium states that were extracted from CDF data [65], Cho predicted that the branching fraction in (26) should be 4.1×10^{-5} . Given the large uncertainties in the color-octet matrix elements, this is in reasonable agreement with the OPAL result (26). If one can show that the same values of the color-octet matrix elements explain Υ production in $p\bar{p}$ colliders and in Z^0 decay, this would provide strong support for the proposal that the production is dominated by color-octet mechanisms.

5.3 Production of the B_c

In the Standard Model, the only bound states consisting of two heavy quarks with different flavors are the $\bar{b}c$ mesons. The ground state of the $\bar{b}c$ system is the pseudoscalar state B_c . The mass spectrum of the $\bar{b}c$ mesons can be predicted reliably from quark potential models that are tuned to reproduce the spectra of charmonium and bottomonium [74]. According to the potential-model calculations, the first two sets of S-wave states, the first and probably the entire second set of P-wave states, and the first set of D-wave states all lie below the BD flavor threshold. Since QCD interactions are diagonal in flavors, the annihilation of $\bar{b}c$ mesons can only occur through a virtual W^+ and is therefore suppressed relative to the electromagnetic and hadronic transitions to lower-lying $\bar{b}c$ states. Thus all the excited states below the BD threshold will cascade down to the ground state B_c via emission of photons and/or pions. A calculation of the inclusive production of the B_c meson must therefore include the contributions from production of all the S-wave, P-wave, and D-wave states that are below the BD threshold.

The production of the B_c and the lowest 3S_1 state B_c^* in e^+e^- annihilation was first computed to leading order in α_s in the color-singlet model by Clavelli and by Amiri and Ji [75]. The lowest-order process for

B_c production is $e^+e^- \rightarrow B_c + b\bar{c}$. Chang and Chen [29] showed that the energy distributions for direct B_c and B_c^* production could be expressed in terms of the fragmentation functions $D_{\bar{b} \rightarrow B_c}(z)$ and $D_{\bar{b} \rightarrow B_c^*}(z)$. For example, the production rate for B_c at the Z^0 resonance in the scaling limit $M_Z \rightarrow \infty$ with $z = 2E_{B_c}/M_Z$ fixed has the form

$$\frac{d\Gamma}{dz}(Z^0 \rightarrow B_c(z) + X) \approx \Gamma(Z^0 \rightarrow b\bar{b}) D_{\bar{b} \rightarrow B_c}(z). \quad (27)$$

The fragmentation functions $D_{\bar{b} \rightarrow B_c}(z)$ and $D_{\bar{b} \rightarrow B_c^*}(z)$ were also calculated later by Braaten, Cheung, and Yuan [28], who pointed out that they were universal and could also be used to calculate direct B_c production in other high-energy processes, such as $p\bar{p}$ collisions. The first calculations of B_c production in hadron colliders using fragmentation functions were carried out by Cheung [76]. The fragmentation functions were subsequently calculated for the P-wave states of the $\bar{b}c$ system [32, 33] and even for the D-waves [36], allowing a complete calculation of the inclusive B_c production rate. The advantage of the fragmentation approach over carrying out a full calculation

of the production rate to leading order in α_s is that the calculation of the fragmentation function is much simpler and it can be extended to P-wave and D-wave states relatively easily. The limitation of the fragmentation approach is that it is accurate only at sufficiently large p_T .

Color-octet production mechanisms are expected to be much less important for $\bar{b}c$ mesons than for charmonium or bottomonium. All terms in the gluon fragmentation functions are of order α_s^3 or higher. For the b -quark fragmentation functions, the leading color-singlet terms and the leading color-octet terms are both of order α_s^2 . For S-wave states like B_c and B_c^* , the color-octet terms are therefore suppressed relative to the color-singlet terms by a factor of v^4 from the ratio of the NRQCD matrix elements. For P-wave states, a color-octet term in the fragmentation function is necessary to avoid an infrared divergence in the color-singlet fragmentation function. However, this divergence occurs first at next-to-leading order in α_s . Thus the color-octet term is not as important as it is for the case of gluon fragmentation into χ_c , where the divergence occurs at leading order.

The production of the B_c and B_c^* at hadronic colliders like the Tevatron and the Large Hadron Collider (LHC) has recently been calculated by several groups, both through complete $\mathcal{O}(\alpha_s^4)$ calculations [77, 78] and using the simpler fragmentation approximation [76, 79]. Discrepancies among the earlier calculations raised questions about the accuracy

of the fragmentation approximation in the $\bar{b}c$ case. Recent calculations [78] have shown that the fragmentation approximation is very accurate for the B_c for p_T as low as 10 GeV. However, for the B_c^* , the accuracy is much lower, decreasing to 20% only for p_T around 30 GeV. For B_c production in $\gamma\gamma$ collisions, even larger values of p_T are required before the fragmentation approximation becomes accurate [80]. The fact that such large values of p_T are required in order for fragmentation to dominate can be attributed to the presence of an additional small parameter m_c/m_b in the problem. Contributions that fall off asymptotically as m_b^2/p_T^2 may dominate at sub-asymptotic values of p_T if they are enhanced by powers of m_b/m_c .

The only calculations presently available for the production of the P-wave $\bar{b}c$ mesons have been carried out using the fragmentation approximation. This approach has been used by Cheung and Yuan to calculate the inclusive production of the B_c in $p\bar{p}$ collisions, including the contributions from all the S-wave and P-wave states below the BD threshold [79]. With acceptance cuts of $p_T > 6$ GeV and $|y| < 1$, the inclusive production cross section for the B_c meson at the Tevatron is about 5 nb. The contributions from the excited S-wave states and from the P-wave states are 58% and 23%, respectively, and hence significant. The D-wave states were not included in this analysis since they are expected to contribute only about 2% to the inclusive production of B_c [36]. The search for the B_c is now underway at the Tevatron, and preliminary results have already been presented by the CDF collaboration [81]. If the B_c is not found in the present run of the Tevatron, it should certainly be discovered after the installation of the Main Injector, which will boost the luminosity by about a factor of 10.

The B_c may also be discovered at LEP. Candidate events for the B_c have already been reported by the ALEPH collaboration [82]. The branching fraction predicted from the color-singlet processes $Z^0 \rightarrow B_c + b\bar{c}$ and $Z^0 \rightarrow B_c^* + b\bar{c}$ are 5.9×10^{-5} and 8.3×10^{-5} , respectively [28]. If the fragmentation functions are used to estimate the contributions of higher S-wave, P-wave, and D-wave states, the inclusive branching fraction for B_c is predicted to be larger by about a factor of 5.

6 OUTLOOK

The NRQCD factorization formalism provides a very general framework for analyzing the production of heavy quarkonium. It implies that the production process is much more complex than has been assumed in the

color-singlet model. There are infinitely many nonperturbative matrix elements that contribute to the cross sections, although only a finite number of them contribute at any given order in v^2 . The most dramatic consequence of this formalism is that color-octet mechanisms sometimes give the largest contributions to the cross section. In the case of P-waves, there is a color-octet term at leading order in v^2 that must be included for perturbative consistency. In the case of S-waves, color-octet terms are suppressed by v^4 , but they may be important if the color-singlet term is suppressed by other small parameters such as α_s .

The NRQCD factorization formalism suggests an explanation for the large production rates for bottomonium and prompt charmonium that have been observed at the Tevatron. They can be attributed to color-octet terms in the cross section that are important because the color-singlet terms are suppressed by powers of α_s and m_Q^2/p_T^2 . The magnitudes of the color-octet matrix elements that are required to explain the Tevatron data are in accord with expectations from NRQCD. This explanation leads to many predictions that can be tested experimentally. The ultimate test is that the same matrix elements must be able to explain heavy quarkonium production in other high energy processes, such as Z^0 decay.

In the NRQCD factorization formalism, there are many nonperturbative matrix elements that must be determined phenomenologically in order to make theoretical predictions for quarkonium production. Fortunately, there is a wealth of experimental data that can be used to determine these matrix elements. Quarkonium is produced as a byproduct of almost every high energy experiment. In this review, we have only discussed data from the highest-energy colliders, the Tevatron and LEP. However there is also data on quarkonium production from lower-energy e^+e^- and ep colliders. There is also an abundance of data on pN , πN , and γN collisions from fixed target experiments. By carrying out a comprehensive analysis of all the data on quarkonium production, it should be possible to determine most of the NRQCD matrix elements that are of phenomenological importance.

An enormous amount of theoretical work will be required in order to carry this program to completion. For every process, the production rate should be calculated to next-to-leading order in all the small parameters in the problem, including v^2 , α_s , and kinematical parameters such as m_Q^2/p_T^2 . Most calculations until very recently were carried out within the color-singlet model, and therefore include only contributions that are of leading order in v^2 . Even in the color-singlet model, the only process for which the corrections at next-to-leading order in α_s have

been calculated is photoproduction of J/ψ [83]. The effort to calculate contributions that are beyond leading order in v^2 , such as color-octet production mechanisms, has just begun.

While the NRQCD factorization approach provides a very general framework for analyzing quarkonium production, it should not be regarded as a complete theory. The derivation of the factorization formula breaks down when there are hadrons in the initial state if the quarkonium is produced with small transverse momentum. Therefore, processes involving diffractive scattering, such as the elastic scattering process $\gamma p \rightarrow \psi p$, are not described by this formalism. The NRQCD formalism also is not appropriate for describing the formation of charmonium from intrinsic $c\bar{c}$ pairs that come from the parton distribution of a colliding hadron [84].

The complications of diffractive scattering and intrinsic $c\bar{c}$ pairs do not arise in the production of heavy quarkonium at large p_T , and therefore the theoretical analysis of this process is particularly clean. The factorization theorems of perturbative QCD guarantee that the production is dominated by fragmentation, the formation of heavy quarkonium within “jets” that are initiated by single high-energy partons. The large cross sections for prompt charmonium production that have been observed at the Tevatron can be explained by including a color-octet term in the gluon fragmentation function. While the normalization of the cross section is not predicted, this mechanism does give other predictions that can be tested experimentally. The most dramatic prediction is a large transverse spin alignment for direct ψ and ψ' at large p_T , but the spin alignment has not yet been measured. This mechanism also gives predictions for the correlations between high- p_T charmonium states and other jets produced by the collision that can be tested.

An enormous amount of theoretical work remains to be done in order to obtain precise predictions for fragmentation that can be compared with experiment. Quantitative estimates of all the relevant NRQCD matrix elements are required in order to determine which terms in the fragmentation functions are numerically important. The important terms should all be calculated to next-to-leading order in α_s . It is also important to calculate the power corrections to the cross sections that fall as m_Q^2/p_T^2 , since they may give the largest corrections to some observables.

New experimental data from high-energy colliders is providing stringent tests of our understanding of the production of heavy quarkonium. The NRQCD factorization approach suggests that cross sections should be calculable in terms of the heavy-quark mass m_Q , the running coupling constant α_s , and a few matrix elements that can be determined

phenomenologically. This approach provides an explanation for the large production rates that have been observed at the Tevatron and at LEP, but further effort, both theoretical and experimental, will be required to show conclusively that this explanation is correct. If this effort is successful, it will mark a major milestone on the road to a comprehensive description of heavy-quarkonium production in all high energy processes.

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Figure Captions

Figure 1 CDF data on the differential cross section for prompt ψ 's that do not come from χ_c decay as a function of p_T . The curves are the leading-order predictions of the color-singlet model (dashed curve), the predictions from fragmentation in the color-singlet model (dotted curve), and the contribution from gluon fragmentation via the color-octet mechanism (solid curve) with the normalization adjusted to fit the CDF data.

Figure 2 CDF data on the differential cross section for prompt ψ 's from χ_c decay as a function of p_T . The dashed and solid curves are as described in Figure 1.

Figure 3 CDF data on the differential cross section for prompt ψ 's as a function of p_T . The curves are as described in Figure 1.

Figure 4 Predictions for the energy spectrum $d\Gamma/dz$ for prompt ψ 's from Z^0 decay in the color-singlet model (dashed line) and from the leading color-octet process (solid line).

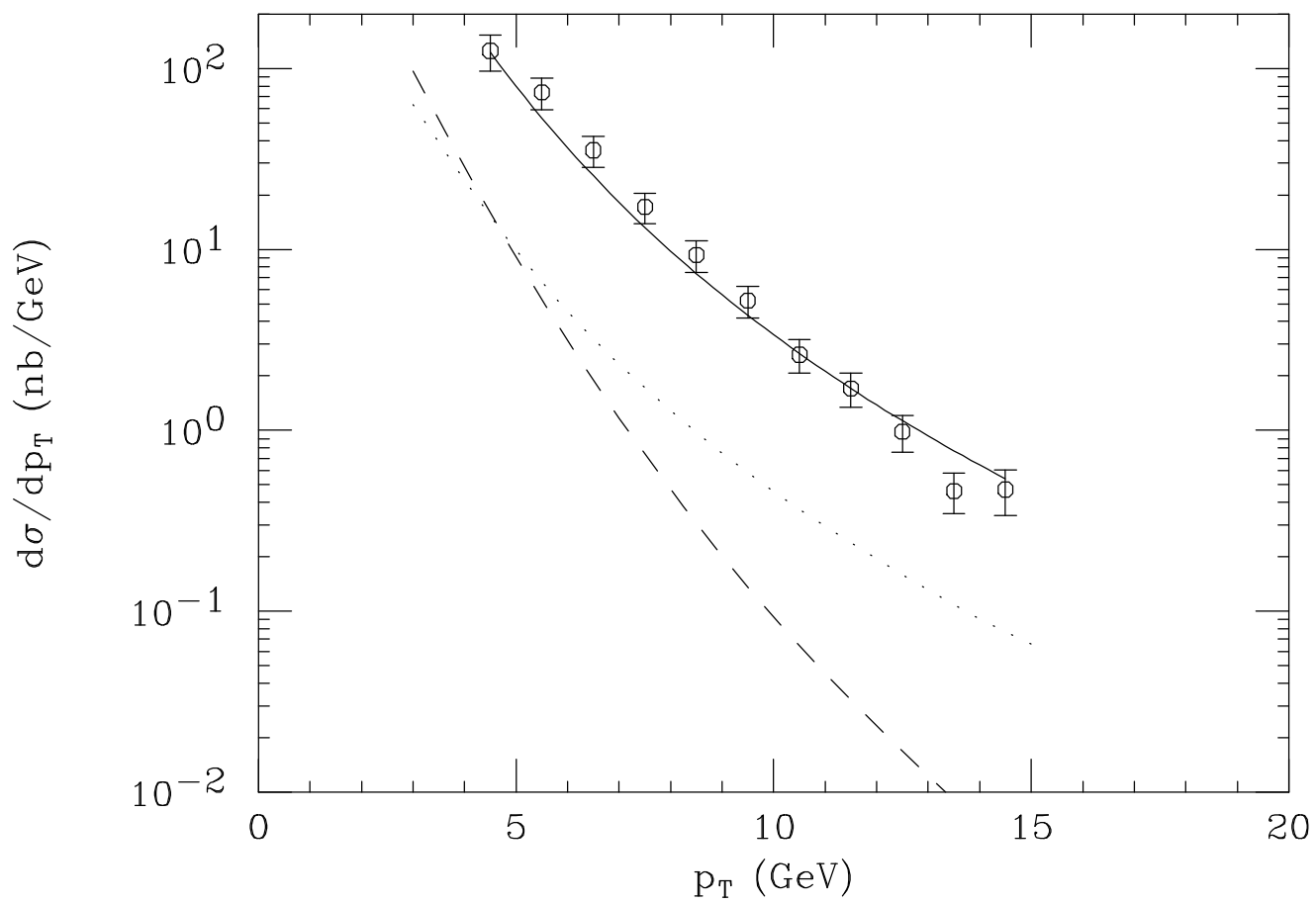


Figure 1

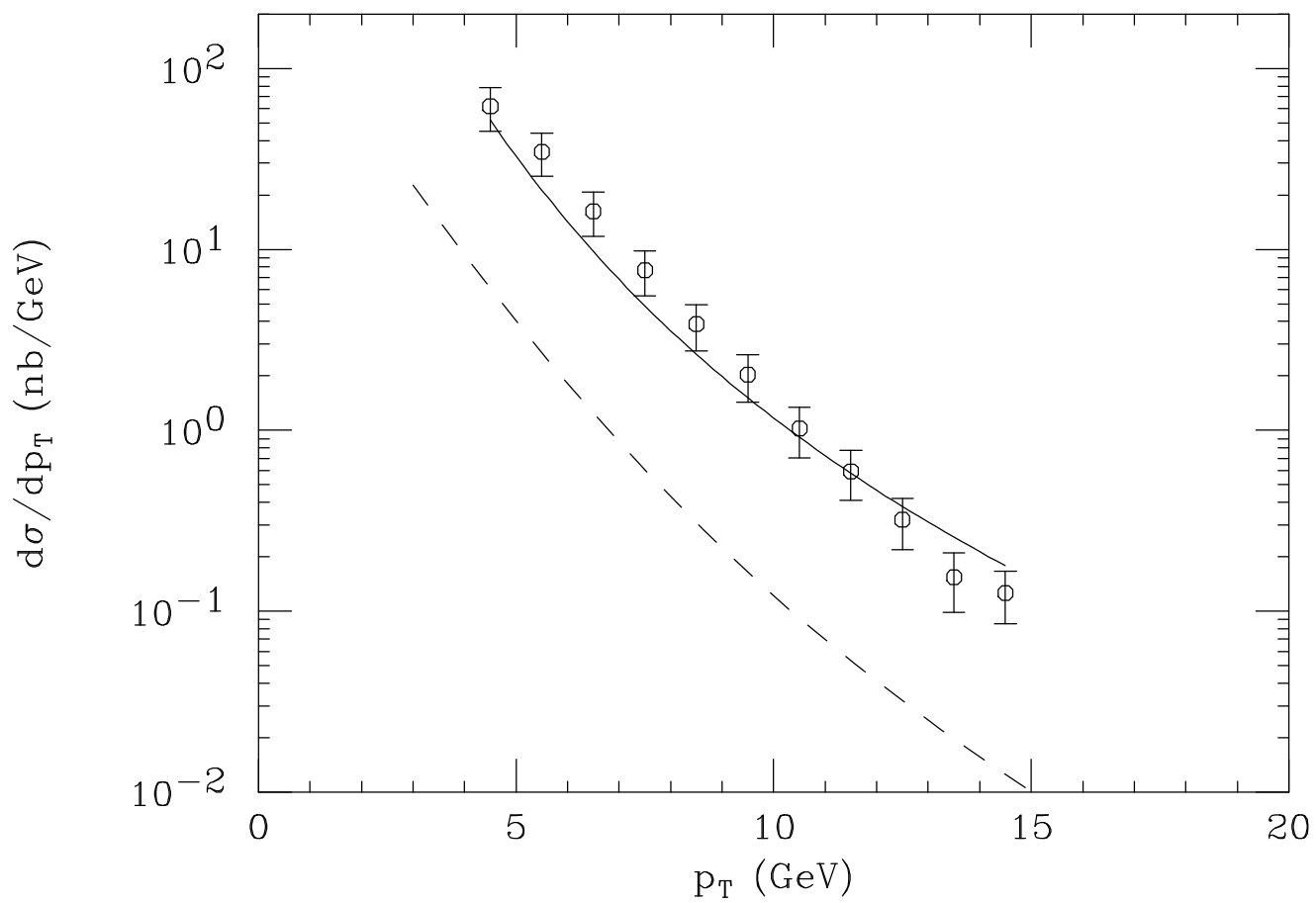


Figure 2

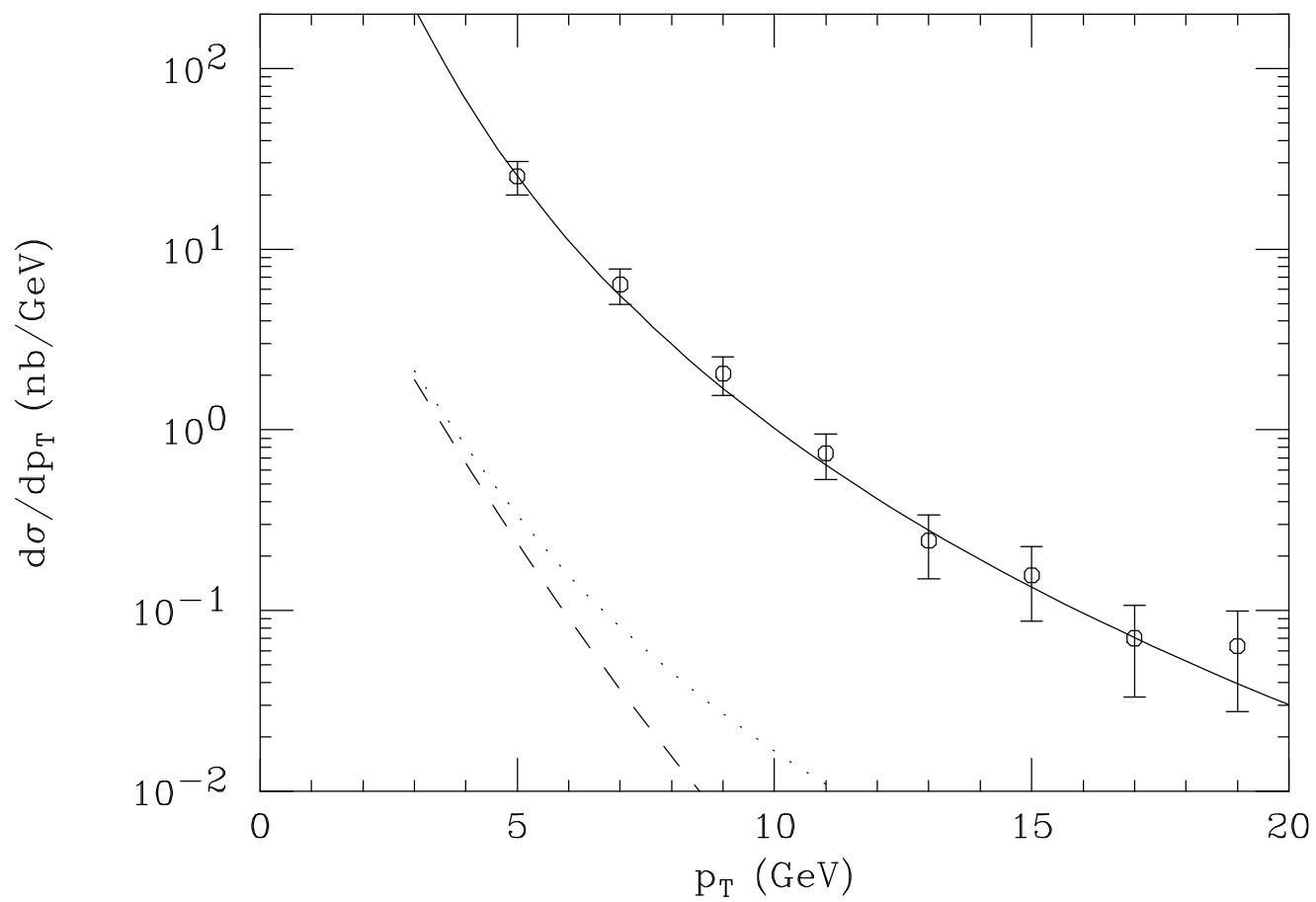


Figure 3

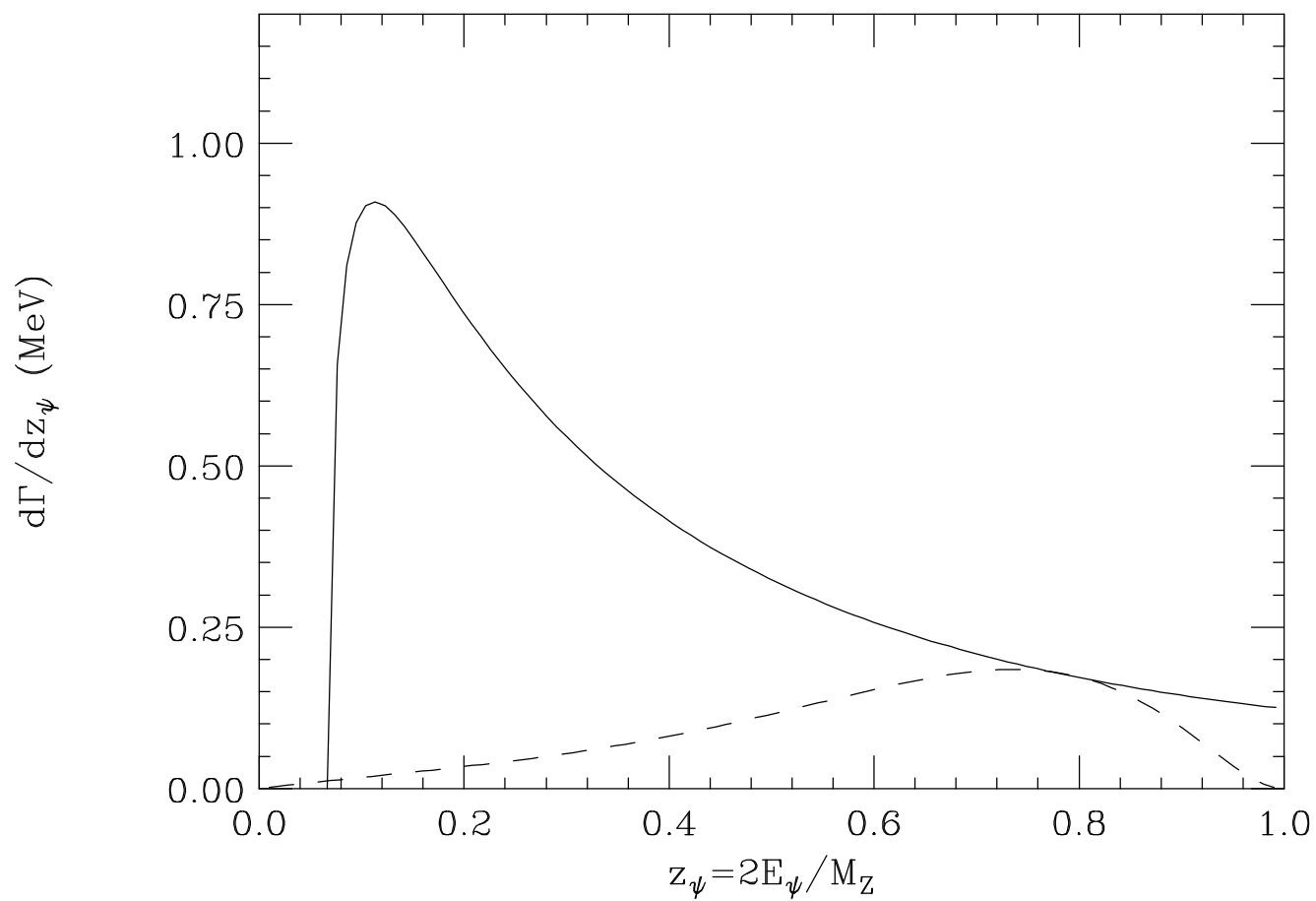


Figure 4